chapter-~~3~~2 (New syllabus)

Topic: Motion In A Straight Line   Date

Example ~~3.2~~ 2.1 The position of - - - - - - - - - and $t = 4\,s$?

Solution: Position of the object is given by

$$x = a + bt^2$$

where $a = 8.5\ m$ and $b = 2.5\ m/s^2$

We know

$$V = \frac{dx}{dt}$$

$$= \frac{d}{dt}(a + bt^2)$$

or $V = 0 + 2bt$ $\left[\because \frac{d}{dt} t^n = n t^{n-1}\right]$

or $V = 2bt$

Now at $t = 0$, $V = 0$ Ans

and at $t = 2.0\,s$

$$V = 2b \times 2$$

$$= 4b$$

$$= 4 \times 2.5 \quad [\because b = 2.5\,m/s^2]$$

$$V = 10\ m/s$$ Ans

For average velocity

by $x = a + bt^2$

displacement at $t = 2.0$ sec

$$x_1 = a + b(2)^2 = a + 4b$$

displacement at = $t = 4.0$ sec

$$x_2 = a + b(4)^2 = a + 16b$$

We know

$$V_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{a + 16b - (a + 4b)}{4 - 2}$$

or $V_{av} = \frac{12b}{2} = 6b$

or $V_{av} = 6 \times 2.5 = 15\ m/s$ Ans

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Example 2.2

Calculus Method: $u \to$ velocity at $t=0$, $v \to$ velocity at time 't'

(i) First Equation:

We have, $a = \frac{dv}{dt}$

$$dv = a\,dt$$

$$\int_u^v dv = \int_0^t a\,dt$$

$$[v]_u^v = a[t]_0^t$$

$$v - u = a(t-0)$$

$$\boxed{v = u + at}$$

$\frac{d}{dx}x^n = nx^{n-1}$

$v = \frac{dx}{dt}$, $a = \frac{dv}{dt}$

$\int x^n dx = \frac{x^{n+1}}{n+1}$

$\int dx = x$, $\int dv = v$

$\int dt = t$, $\int t\,dt = \frac{t^2}{2}$

$\int_u^v v\,dv = \left[\frac{v^2}{2}\right]_u^v = \frac{v^2-u^2}{2}$

(ii) Second Equation

We know, $v = \frac{dx}{dt}$

$$dx = v\,dt$$

$$dx = (u+at)\,dt$$

or $$dx = u\,dt + at\,dt$$

On integrating within the limits,

$$\int_{x_0}^{x} dx = \int_0^t u\,dt + \int_0^t at\,dt$$

or $$[x]_{x_0}^{x} = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

or $$x - x_0 = u(t-0) + a\left(\frac{t^2}{2} - 0\right)$$

put $x - x_0 = S$,

$$\boxed{S = ut + \frac{1}{2}at^2}$$

(III) Third Equation

We know, $a = \frac{dv}{dt}$

$$= \frac{dv}{dt} \cdot \frac{dx}{dx}$$

$$= \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$a = \frac{dv}{dx} \cdot v$$

or $$a = v\frac{dv}{dx}$$

or $$v\,dv = a\,dx$$

On integrating within the limit,

$$\int_u^v v\,dv = \int_{x_0}^{x} a\,dx$$

$$\left[\frac{v^2}{2}\right]_u^v = a\left[x\right]_{x_0}^{x}$$

or $$\frac{v^2}{2} - \frac{u^2}{2} = a(x - x_0)$$

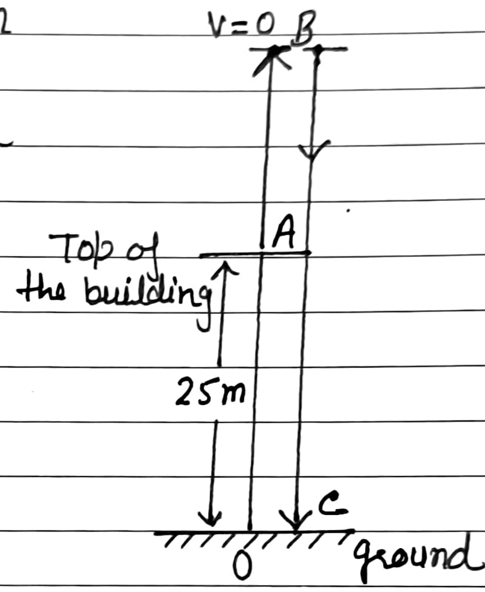
put $x - x_0 = s$

$$\frac{v^2 - u^2}{2} = as$$

or $$\boxed{v^2 - u^2 = 2as}$$

Example ~~3.4~~ 2.3 A ball is thrown - - - - - hits the ground?
Take $g = 10\ m/s^2$

Solution The ball is thrown from the top of the building with initial velocity
$u = 20\ m/s$
height of the building $h = 25\ m$
and $g = 10\ m/s^2$



(a) by III eq^n of motion

$$v^2 = u^2 + 2gh$$

at maximum height (At B)
$v = 0$.

then
$$0 = 20^2 + 2(-10)h \qquad [\because g = -10\ m/s^2]$$

or $h = \dfrac{20 \times 20}{20}$

or $h = 20\ m$ Ans

(b) Net displacement AC $= S = -25\ m$
-ve sign is taken because displacement is in the opposite direction of initial velocity.

by $S = ut + \frac{1}{2} g t^2$

$$-25 = 20t + \frac{1}{2}(-10)t^2$$

or $5t^2 - 20t - 25 = 0$

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or $t^2 - 4t - 5 = 0$
or $(t-5)(t+1) = 0$
or $t = 5$ sec as $t \neq -1$

Therefore time taken to hit the ground = 5 sec
Ans

2.4
Example ~~3.5~~ Discuss the motion - - - - - - - - air resistance.

Solution: We know that for verticle motion, Equations of motion become:

(i) $v = u + gt$
(ii) $h = ut + \frac{1}{2}gt^2$
(iii) $v^2 = u^2 + 2gh$

Here we assume that motion is in y-direction. The object is released from rest at $y = 0$. Therefore we take $-y$ direction and $u = 0$ also we take $a = -g \;(= -9.8\ m/s^2)$

Hence the eqn's become
(i) $v = 0 - gt$
$v = -gt \Rightarrow \boxed{v = -9.8t\ m/s}$
(ii) $y = 0 - \frac{1}{2}gt^2$
or $y = -\frac{1}{2}gt^2 \Rightarrow \boxed{y = -4.9t^2\ m}$
(iii) $v^2 = 0 - 2gy$
or $v^2 = -2gy \Rightarrow \boxed{v^2 = -19.6y\ m^2/s^2}$

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Example ~~3.6~~ 2.5

Galileo's law of odd numbers: The distances traversed - - - - - - - Prove it.

Solution:
Let us divide the time interval of motion of an object under free fall into many equal intervals.
We know for free fall

$$y = -\frac{1}{2} g t^2 \qquad [\because u = 0]$$

$$\text{or} \quad y = -4.9 t^2 \qquad [\because g = 9.8 m/s^2]$$

by using this relation we find the distances after 1s, 2s, 3s - - - in terms of y_0 which is distance travelled in 1st second.
We write these values in a table -

t (sec)	$y(= -4.9t^2)$	distance traversed in successive interval	Ratio
$t=0$	$y=0$	-	-
$t=1$	$y=-4.9 = \underline{y_0}$ (say)	$y_0 - 0 = y_0$	1
$t=2$	$y = -4.9 \times 2^2 = -4 \times 4.9 = \underline{4y_0}$	$4y_0 - y_0 = 3y_0$	3
$t=3$	$y = -4.9 \times 3^2 = -9 \times 4.9 = \underline{9y_0}$	$9y_0 - 4y_0 = 5y_0$	5
$t=4$	$y = -4.9 \times 4^2 = -16 \times 4.9 = \underline{16y_0}$	$16y_0 - 9y_0 = 7y_0$	7
$t=5$	$y = -4.9 \times 5^2 = -25 \times 4.9 = \underline{25y_0}$	$25y_0 - 16y_0 = 9y_0$	9

From the table it is clear that the distances are in the simple ratio 1 : 3 : 5 : 7 : 9 : 11 - - - -

Proved

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II. Method

The distance traversed in nth second by a freely falling body is given by

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

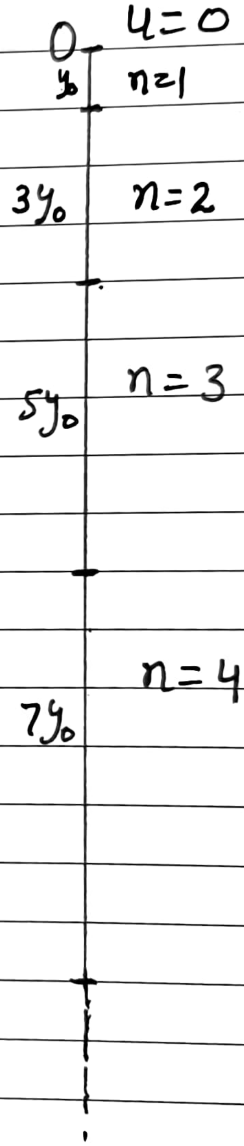
For free fall

$$u = 0 \quad \text{and} \quad a = g = 9.8\ m/s^2$$

so

$$S_{nth} = \frac{9.8}{2}(2n-1)$$

or $$S_{nth} = 4.9(2n-1)$$



Now for $n=1$

$$S_1 = 4.9(2\times1-1)$$

or $$S_1 = 4.9 = y_0 \text{ (say)}$$

for $n=2$

$$S_2 = 4.9(2\times2-1)$$

or $$S_2 = 3\times4.9 = 3y_0$$

for $n=3$

$$S_3 = 4.9(2\times3-1)$$

or $$S_3 = 5\times4.9 = 5y_0$$

similarly

$$S_4 = 7\times4.9 = 7y_0$$

$$S_5 = 9\times4.9 = 9y_0$$

$$S_6 = 11\times4.9 = 11y_0$$

It is clear that

$$S_1 : S_2 : S_3 : S_4 : S_5 \cdots\cdots = 1:3:5:7:9 \cdots$$

Proved

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Example ~~3.7~~ 2.6

Initial velocity of vehicle $= v_0$
Deceleration $= -a$
Final velocity $v = 0$

Let the stopping distance is d_s
We have III. eqn of motion

$$v^2 = v_0^2 + 2aS$$
$$0 = v_0^2 + 2(-a) d_s$$

$$\text{or} \quad d_s = \frac{v_0^2}{2a}$$

If $v_0 \to 2v_0$
then $d_s \to 4d_s$

Example ~~3.8~~ 2.7

Given $h = d = 21\ cm = 0.21\ m$
$t = ?\ ,\ u = 0$

by $h = ut + \frac{1}{2} g t^2$

$$0.21 = \frac{1}{\cancel{2}} \times \cancel{9.8}^{4.9} t^2$$

$$\text{or} \quad t^2 = \frac{0.21}{4.9}$$

$$t = \sqrt{\frac{\cancel{0.21}\,0\,30}{\cancel{4.9}_{7} \times 100}} = \sqrt{\frac{30}{7 \times 100}}$$

$$= \frac{\sqrt{4.3}}{10} \approx \frac{2}{10} = 0.2\ \text{Sec}$$

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R.T. $t = 0.2S$

X (Not in new syllabus)

Example 3.9: Two parallel rail - - - - - - on the ground?

Solution: Given,

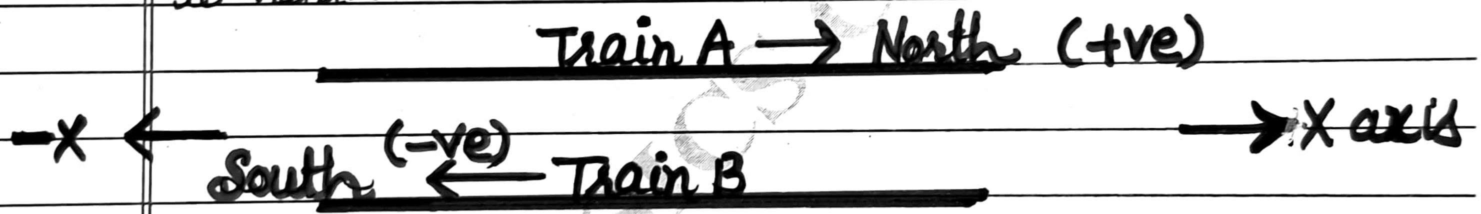
Speed of train A $V_A = 54 \text{ Km/H}$

$= \frac{54 \times 5}{18} = 15 \text{ m/s}$

Speed of train B $V_B = 90 \text{ Km/H}$

$= \frac{90 \times 5}{18} = 25 \text{ m/s}$

Train A moves north and train B moves south. Let positive direction of x-axis is south to north.



(a) Velocity of B with respect to A

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$= -25 - 15 \qquad [\because \vec{V}_B = -25 \text{ m/s}]$

$= -40 \text{ m/s}$ Ans.

The train B appears to A to move with a speed of 40 m/s from the north to south.

(b) Relative velocity of ground with respect to B

$\vec{V}_{GB} = \vec{V}_G - \vec{V}_B$

$\vec{V}_{GB} = 0 - (-25)$ $\quad [\because V_G = 0]$

$= 25\ m/s$ Ans

i.e the ground appears to B to move with a speed of 25 m/s from south to north.

(C) Relative velocity of monkey with respect to the train A

$\vec{V}_{MA} = -18\ Km/H$

$= -18 \times \frac{5}{18} = -5\ m/s$

$\vec{V}_A = 15\ m/s$ and $\vec{V}_M = ?$

$\vec{V}_{MA} = \vec{V}_M - \vec{V}_A$

$-5 = \vec{V}_M - 15$

or $\vec{V}_M = 15 - 5$

or $\vec{V}_M = 10\ m/s$ Ans

i.e the velocity of monkey with respect to ground $V_M = 10\ m/s$

Monkey will appear to move from south to north.