Formula Sheet

Ch. 12

Atoms and Nuclei

1. Rutherford's α particle scattering experiment (Geiger-Marsden experiment)

No. of α particles scattered

$$N \propto \frac{1}{\sin^4\frac{\theta}{2}}$$

$\theta \rightarrow$ angle of scattering

2. Distance of closest approach: Estimation of size of nucleus

$$r_0 = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{\frac{1}{2}mv^2} = \frac{k\,2Ze^2}{E_K}$$

coulomb's constant $\left(\frac{1}{4\pi\varepsilon_0} = k\right)$

$Z \rightarrow$ Atomic no. of nucleus (= no. protons in atom)

$E_K \rightarrow$ Kinetic energy

$e \rightarrow$ charge of electron

3. Impact parameter (b)

$$b = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2\cot(\theta/2)}{E_K}$$

$\left[E_K = \frac{1}{2}mv^2\right]$

$\theta \rightarrow$ angle of scattering

4. Bohr's atomic model

- Angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$

$$mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$$

where $n = 1, 2, 3 \ldots$ (principle quantum number)

- Energy of emitted photon

$$h\nu = E_2 - E_1$$

E_1 and E_2 are energies of eletron in orbits

5. Radius of orbit of electron

$$r = \frac{n^2 h^2}{4\pi^2 m k Z e^2}$$

$\Rightarrow r \propto n^2$
(for hydrogen $Z=1$)
($r_1 : r_2 : r_3 = 1:4:9$)

$n \rightarrow$ principle quantum number
$h \rightarrow$ Planck's constant
$m \rightarrow$ mass of electron
$k \rightarrow \frac{1}{4\pi\epsilon_0}$ coloumb's constant

6. Velocity of electron

$$v = \frac{2\pi k Z e^2}{n h}$$

[for hydrogen $Z=1$]

7. Frequency of electron

$$\nu = \frac{k Z e^2}{n h r} = \frac{4\pi Z^2 e^4 m k^2}{n^3 h^3}$$

$$\nu \propto \frac{Z^3}{n^3}$$

$\left[k = \frac{1}{4\pi\epsilon_0}\right]$

for hydrogen $Z=1$

Kinetic energy of electron in any orbit

$$E_K = \frac{2\pi^2 m e^4 Z^2 k^2}{n^2 h^2}$$

$$E_K = \frac{13.6 Z^2}{n^2} \text{ eV}$$

K.E

$$\frac{mv^2}{r} = \frac{kZe^2}{r^2}$$

or $mv^2 = \frac{kZe^2}{r}$

$$\frac{1}{2}mv^2 = \frac{kZe^2}{2r}$$

$$K.E = \frac{kZe^2}{2r}$$

Potential energy of electron

$$E_p = -\frac{4\pi^2 m e^4 Z^2 k^2}{n^2 h^2}$$

$$E_p = -\frac{27.2 Z^2}{n^2} \text{ eV}$$

$$P.E = -\frac{kZe^2}{r}$$

$$T.E = K.E + P.E$$
$$= \frac{kZe^2}{2r} + \left(-\frac{kZe^2}{r}\right)$$

$$T.E = -\frac{kZe^2}{2r}$$

Total energy of electron

$$E_n = -\frac{2\pi^2 m e^4 Z^2 k^2}{n^2 h^2} = -\frac{13.6 Z^2}{n^2} \text{ eV}$$

For hydrogen atom

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$ (ionisation potential)

Relation b/w kinetic, potential and total energy

- $K.E = -(T.E)$
- $P.E = -2(T.E)$
- $K.E = -\frac{P.E}{2}$

Wavelength of radiation emitted from orbit n_2 to n_1

$$\frac{1}{\lambda} = \frac{2\pi^2 m k^2 e^4 z^2}{ch^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or
$$\boxed{\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]}$$ $\left[\frac{1}{\lambda} = \bar{\nu} \text{ wave no.}\right]$

here $R = 1.097 \times 10^7 m^{-1}$ [Rydberg constant]

Hydrogen Spectrum

(I) Lyman Series (electron jumps from $n=2,3,4\ldots$ to $n=1$)

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right] \quad n = 2, 3, 4 \ldots$$

(ultraviolet region)

(II) Balmer Series (electron jumps from $n=3,4,5\ldots$ to $n=2$)

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \quad n = 3, 4, 5 \ldots$$

(visible region)

(III) Paschen Series (electron jumps from $n=4,5,6\ldots$ to $n=3$)

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right] \quad n = 4, 5, 6 \ldots$$

(Infrared region)

(IV) Bracket Series (electron jumps from $n=5,6,7\ldots$ to $n=4$)

$$\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n^2}\right] \qquad n = 5, 6, 7 \ldots$$

(Infrared region)

(V) Pfund Series (electron jumps from n=6,7,8-- to n = 5)

$$\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n^2}\right]$$

(infrared region)

Nuclei

Atomic mass unit

$1\,u = 1.6 \times 10^{-27}\,kg$

$1\,eV = 1.6 \times 10^{-19}\,J$

$1\,MeV = 1.6 \times 10^{-13}\,J$

$1\,u = 1.6 \times 10^{-27}\,kg = 931\,MeV$

Nuclear Radius

$$R = R_0 A^{1/3}$$

$R_0 = 1.2 \times 10^{-15}\,m$

$A \rightarrow$ Mass number

Nuclear density $\rho = 2.3 \times 10^{17}\,kg/m^3$

(ρ is independent of mass no. A)

Einstein relation $E = mc^2$

Mass defect (Δm)

$$\Delta m = [Z m_p + (A-Z) m_n] - M_A$$

$m_p \rightarrow$ mass of proton (1.00728 amu)

$m_n \rightarrow$ mass of neutron (1.00867 amu)

$Z \rightarrow$ number of protons

$(A-Z) \rightarrow$ number of neutrons

$M_A \rightarrow$ Mass of nucleus

Binding energy (B.E)

$$B.E = \Delta m c^2$$
$$= [\{Z m_p + (A-Z) m_n\} - M_n] c^2$$

Binding energy per nucleon

$$B.E \text{ per nucleon} = \frac{B.E}{A}$$

* Isotopes → Atoms of an element having same atomic number but different mass number.

e.g. → 3 isotopes of Hydrogen

^1_1H ^2_1H ^3_1H

* Isobars → Atoms having the same mass number but different atomic number.

e.g. (i) ^3_1H & ^3_2He (ii) $^{37}_{17}Cl$ and $^{37}_{16}S$

(iii) $^{40}_{20}Ca$ and $^{40}_{18}Ar$

* Isotones → The nuclids having same number of neutrons

e.g (i) $^{37}_{17}Cl$ and $^{39}_{19}K$ (ii) $^{198}_{80}Hg$ and $^{197}_{79}Pu$

$A-Z = 37-17 = 39-19 = 20$