ELECTROMAGNETIC INDUCTION

Initially Electricity and Magnetism were considered separate and unrelated but Oersted experiment and few other established the relation b/w these two.

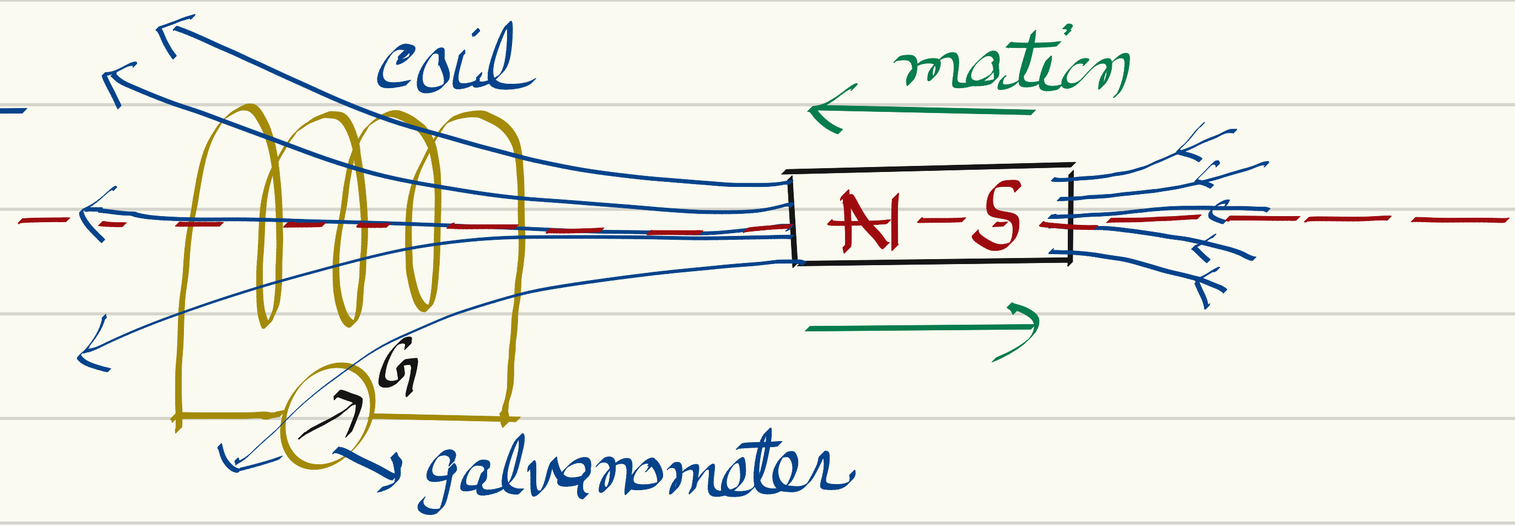
* Moving charges (current) produces magnetic field. Can moving magnets produce electric current?
* Faraday and Henry experiments showed that changing magnetic field link with a coil produces an induced current in the coil.

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Electromagnetic Induction: The phenomenon in which electric current is generated by varying magnetic field is called electromagnetic induction.

The Experiments of Faraday and Henery:

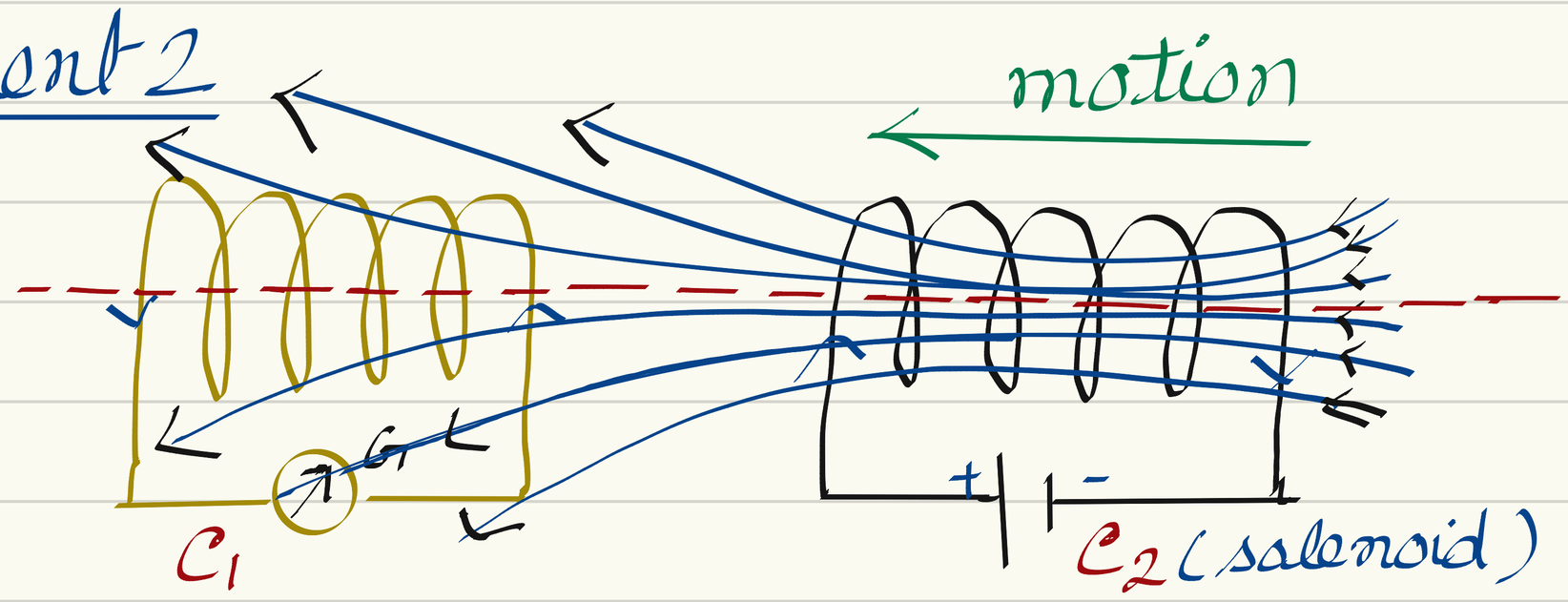
Experiment 1



When the barmagnet is pushed towards the coil, the galvanometer shows deflection.
When magnet is moved away from the coil the galvanometer shows deflection in opposite direction.

Conclusion - It is the relative motion b/w the coil and magnet that is responsible for generation of electric current (induced current) in the coil!

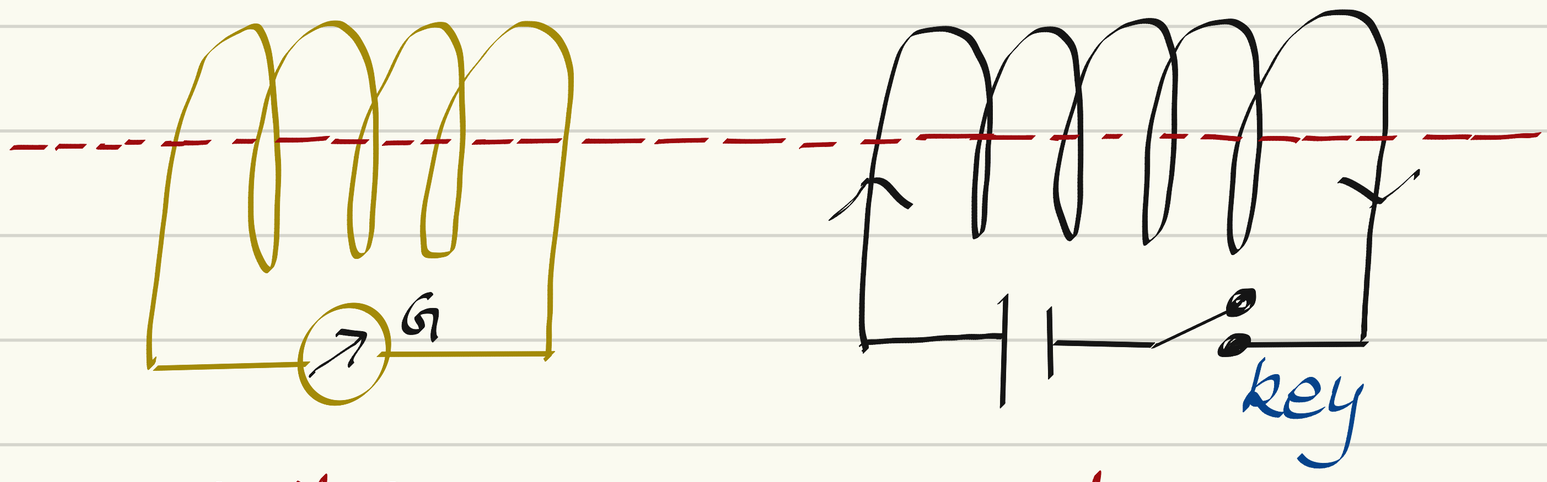
Experiment 2



Current is induced in coil C_1 due to motion of the current carrying coil C_2.

Conclusion - It is the relative motion b/w the coils C_1 and C_2 that induces the electric current.

Experiment 3:



coil C_1 coil C_2

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The two coils held stationary and galvanometer shows momentary deflection when key in coil C_2 is pressed. Deflection is observed again when key is released but in opposite direction.

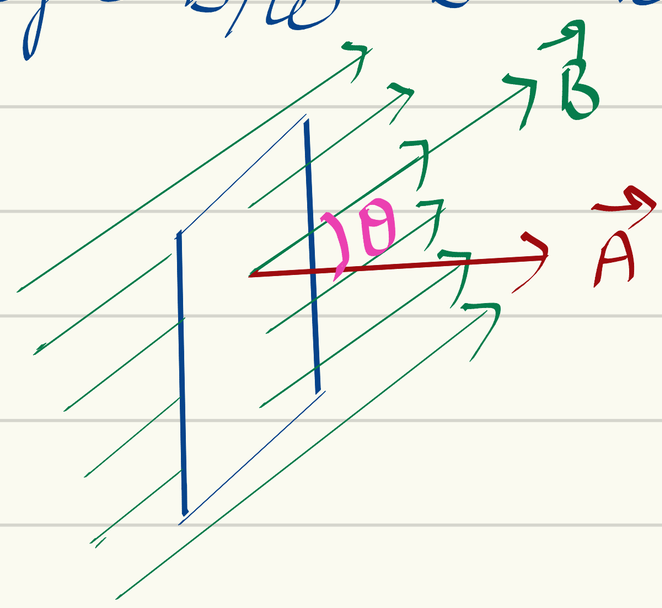
Conclusion - The relative motion b/w the coil is not absolute requirement to generate electric current in coil C_1.

Magnetic Flux: Magnetic flux is the amount of magnetic field passing through a given area.

Magnetic flux through a plane of area A placed in a uniform magnetic field B can be written as

$$\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$$

where θ is angle b/w $\vec{B}$ and $\vec{A}$

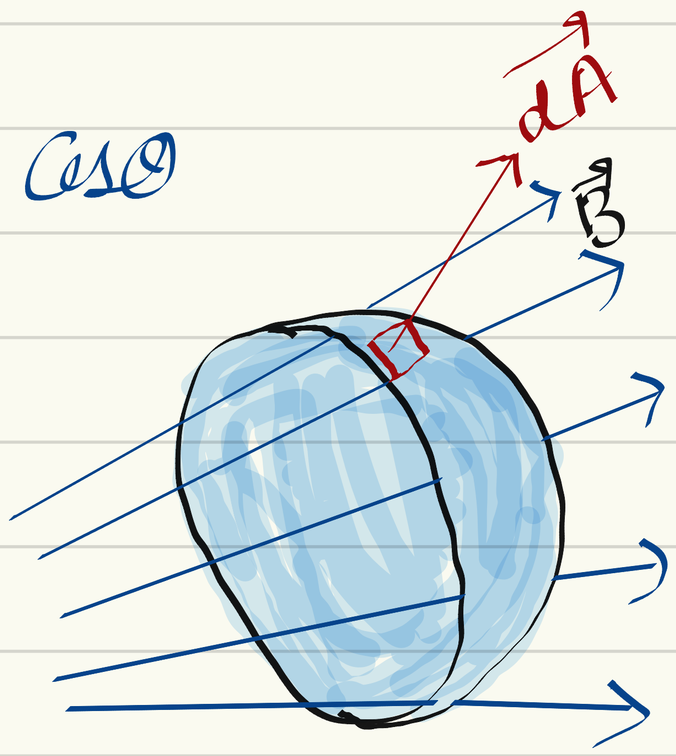


$\vec{A}$ is Area Vector
Dirⁿ → ⊥ to the surface

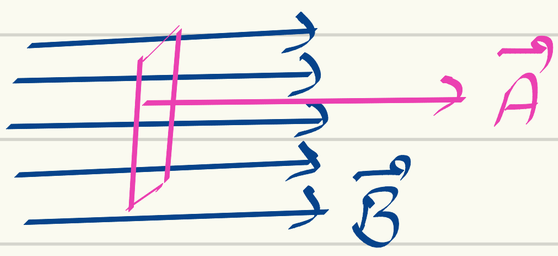
If magnetic field is changing with respect to time then

$$\phi = \int \vec{B} \cdot d\vec{s} = \int B\,ds\cos\theta$$

where ds is area element



* Maximum flux
when $\theta = 0^\circ$



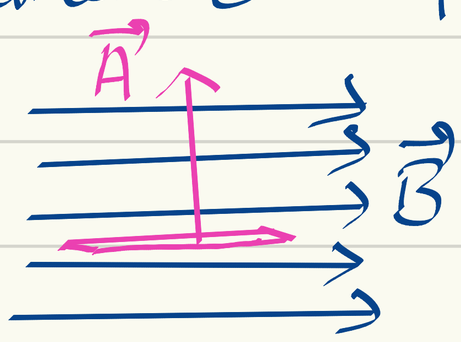
$\phi = BA\cos 0^\circ$

$\boxed{\phi = BA}$ Maximum flux (ϕ_{max})

* Zero flux

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when $\theta = 90^\circ$

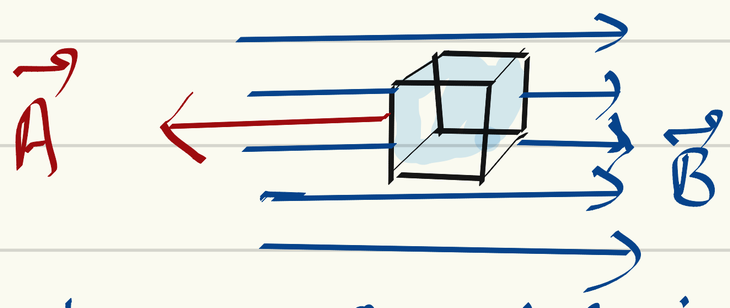


$\phi = BA\cos 90^\circ$
$\phi = 0$ (zero flux)

no magnetic field lines will cross the surface.

* Negative flux

When $\theta > 90$



here $\theta = 180°$

$$\phi = BA\cos 180°$$

$$\phi = -BA \quad (-ve\ flux)$$

* flux is scalar quantity
* SI unit Tm^2 or Weber (Wb)
* Dimensional formula $[ML^2T^{-2}I^{-1}]$

<u>Faraday's Law:</u> The magnetude of induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit.

Mathematically, the induced emf

$$\mathcal{E} = -\frac{d\phi}{dt} \qquad \phi \rightarrow \text{Magnetic flux}$$

For coil of N turns

$$\mathcal{E} = -N\frac{d\phi}{dt}$$

↑ Lenz law

-ve sign indicates the direction of $\mathcal{E}$ and hence the direction of current.

* The induced emf can be increased by increasing the number of turns N of a closed coil.

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Methods of changing Magnetic flux:

(i) By changing B (mag. field)
(ii) By changing A (Area)
(iii) By changing θ (angle b/w B & A)
(iv) By changing the shape of coil (shrinking or stretching the coil)
(v) By rotating a coil in magnetic field such that the angle θ changes.

* In all these cases an emf is induced in the coil.

Lenz Law: According to this law the polarity of induced emf is such that it tends to produced a current which opposes the change in magnetic flux that produced it.

OR

An induced electric current flows in a direction such that the current opposes the change that induced it.

OR

Lenz law states that the direction of current induced in a conductor by changing magnetic field is such that the magnetic field produced by the induced current opposes the change in original magnetic field.

Lenz law formula is derived from Faraday's law of EMI

$$\mathcal{E} = -N\frac{\Delta\phi}{\Delta t} = -N\frac{d\phi}{dt}$$

$\mathcal{E} \rightarrow$ voltage induced
$\Delta\phi \rightarrow$ chang in mag. flux
$N \rightarrow$ no. of turns in coil

Applications of Lenz Law:

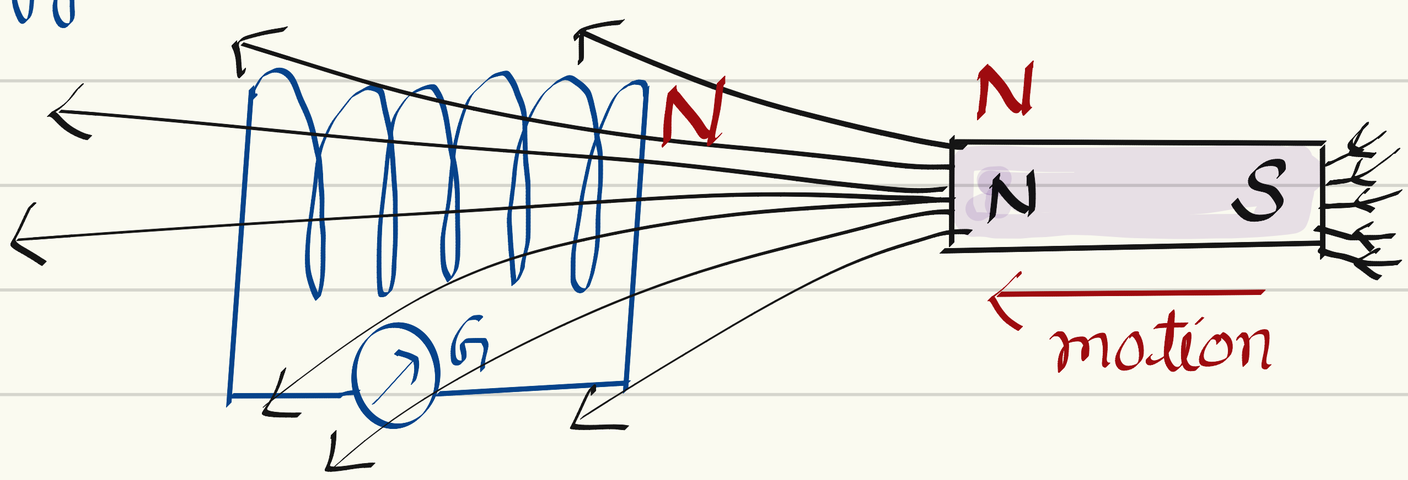
(I) Card reader
(II) Metel detectors
(III) Ac generator
(IV) Microphones
(V) Eddy currents
(VI) Breaking system on trains.

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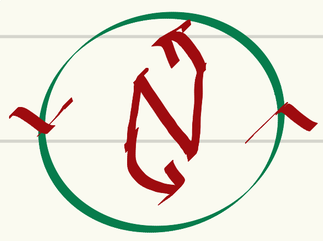
Lenz Law And conservation of Energy:

Lenz law is based upon the law of conservation of energy

(I)

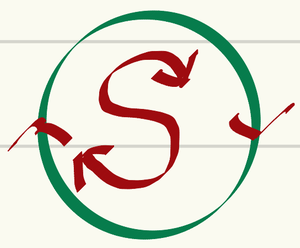


When N-pole of bar magnet is brought near to the coil, magnetic flux liked with the coil increases. According to Lenz law to oppose this increase N-pole is developed on the left side of coil. The direction of induced current is anticlockwise.



Anticlockwise dirⁿ

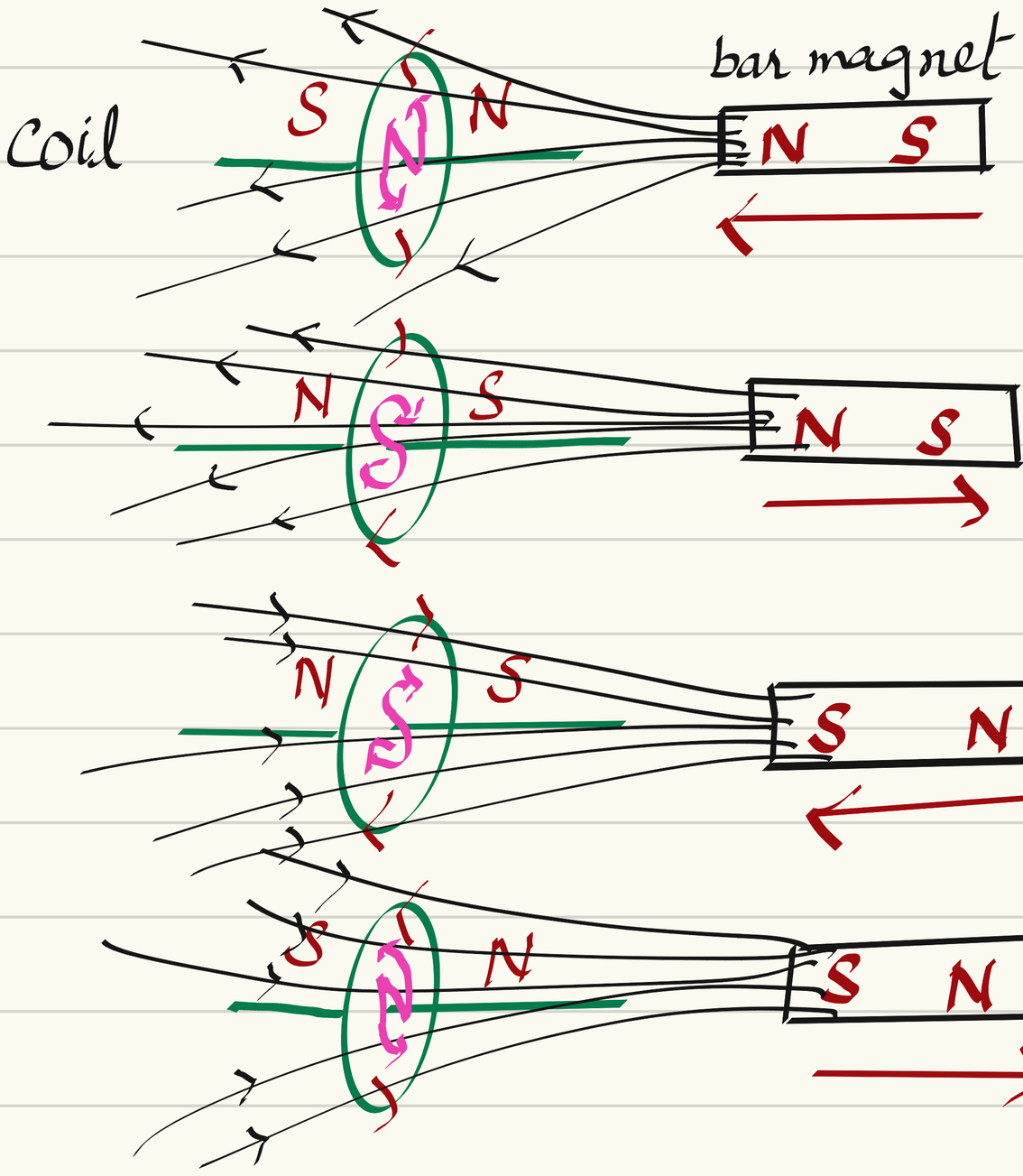
When N-pole is moved away from the coil magnetic flux linked with the coil decreases. To increase the magnetic field S-pole is developed. The dirⁿ of induced current is clockwise.



clockwise dirⁿ

Lenz law does not violate conservation of energy

* All 4 possible cases –

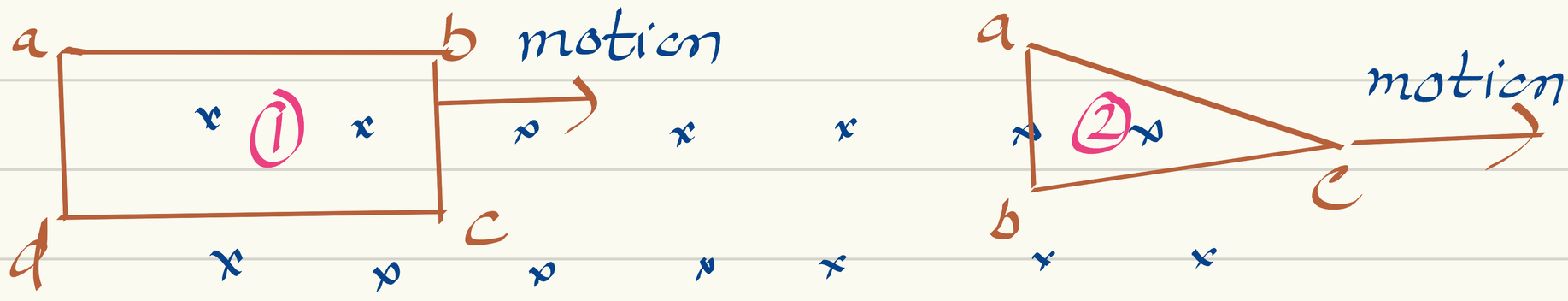


⊗ → cross mag. field → into the page (inwards)

⊙ → dot mag. field → out of the page (outwards)

* As the induced current opposes the change in flux, work has to be done against the opposition offered by induced current. This work appears as electrical energy in the loop. i.e Lenz's is in accordance with the law of conservation of energy

Examples



For loop ①

As loop ① is entering in mag. field B the flux linked with it increasing therefor acc. to Lenz law the induce current will be in such dir^{n} so that it can oppose the

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⊗ → mag. field into the page
⊙ → mag. field out of the page

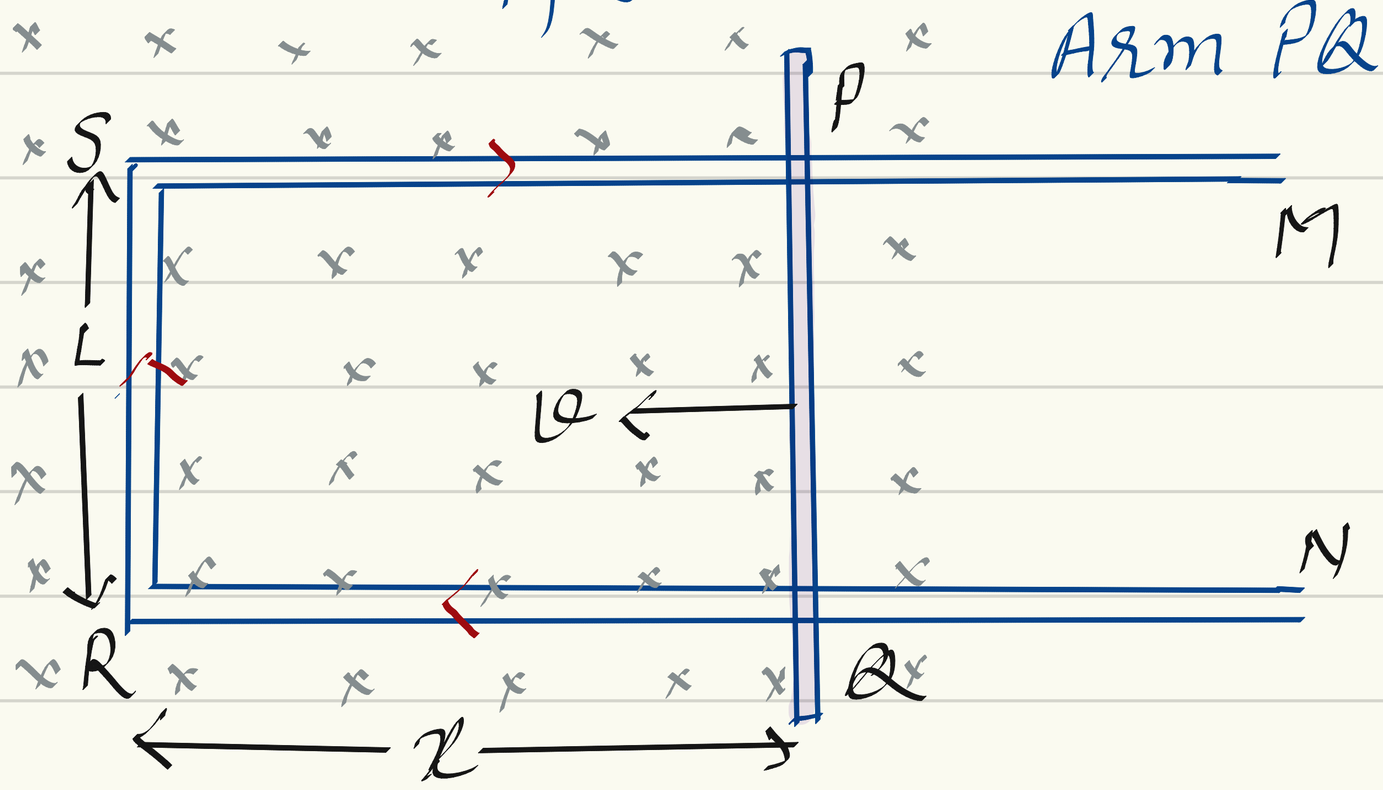
existing field B, which is into the page ⊗. so to create outward field ⊙ dir^n of induced current will be Anticlockwise (ccbad)

For loop ②

Loop ② is moving out from magnetic field so mag. field link with the loop is decreasing. Therefore according to Lenz law to increase the mag. field the dir^n of induced current will be clockwise. so that it can produce mag. field in the same direction of existing field.

Motional Electromotive Force

Consider a straight conductor moving in uniform and time independent magnetic field as shown in fig.



Magnetic flux encloses by the loop PQRS will be

$$\phi = BLx$$

As arm PQ is moving the flux linked with the coil is changing and an induces an emf,

$$\varepsilon = -\frac{d\phi}{dt}$$ [Faraday's law]

$$\mathcal{E} = -\frac{d}{dt}(Blx) \qquad [\phi = Blx]$$

$$= -Bl\frac{dx}{dt}$$

$$\boxed{\mathcal{E} = Blv}$$

Where $\frac{dx}{dt} = -v$ is the speed of conductor PQ.

The induced emf Blv is called motional emf.

Thus induced emf can be produced by moving conductor also.

OR (II method)

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$$\mathcal{E} = \frac{lxl}{q} = \frac{F \cdot dl}{q}$$

each free electron moving within wire experien-ces a force by mag. field. i.e

$$F = \frac{(qvB)l}{q} \qquad [d = l]$$

$$\boxed{F = Blv}$$

* When a conductor is stationary and magnetic field is changing, the force on its charges is given by $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

since $v = 0$ $F = qE$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

Thus the force on charge is due of electric field only.

* Time varying magnetic field generates an electric field.

* Charges in motion (current) can exert force/torque on stationary magnet. conversely, a bar magnet in motion can extert a force on a stationary charge. Which shows Electricity and Magnetism are related.

Emf of A Rotating Rod:

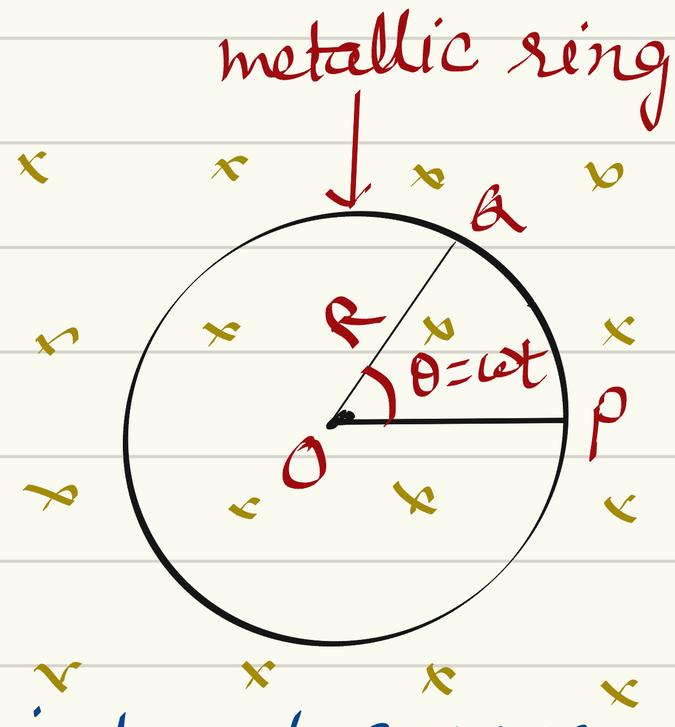


Fig show a rotating rod whose one end is fixed.
A rod rotated electron moves towards outer end due to Lorentz force and get distributed over the ring. Thus and emf is induced across the ends of rod.
The small length of rod dR moves $\perp$ ly to $\vec{B}$

$$d\mathcal{E} = Bvdr \qquad [\because \mathcal{E} = Blv \quad l \to dr]$$

$$\mathcal{E} = \int d\mathcal{E} = \int_0^R Bvdr$$

$$= B\omega \int_0^R r\,dr$$

$$= B\omega \frac{R^2}{2}$$

Or $$\boxed{\mathcal{E} = \frac{1}{2} B\omega R^2}$$

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OR (II Method)

From fig, the area of sector OPQ

$$A = \pi R^2 \times \frac{\theta}{2\pi}$$

$$A = \frac{1}{2} R^2 \theta$$

Now $$\mathcal{E} = \frac{d}{dt}(BA) = B\frac{dA}{dt}$$

Or $$\mathcal{E} = B\frac{d}{dt}\left(\pi R^2 \frac{\theta}{2\pi}\right)$$

$$= \frac{1}{2} B R^2 \frac{d\theta}{dt}$$

or $\boxed{\varepsilon = \frac{1}{2} B R^2 \omega}$ $\left[\because \frac{d\theta}{dt} = \omega\right]$

Inductance - Inductance is the ability to store energy in the form of magnetic field.

An electric current can be induced in a coil by flux change in the nearby coil or in the same coil. In both the cases

$$\phi \propto I$$

also, $$\frac{d\phi}{dt} \propto \frac{dI}{dt}$$

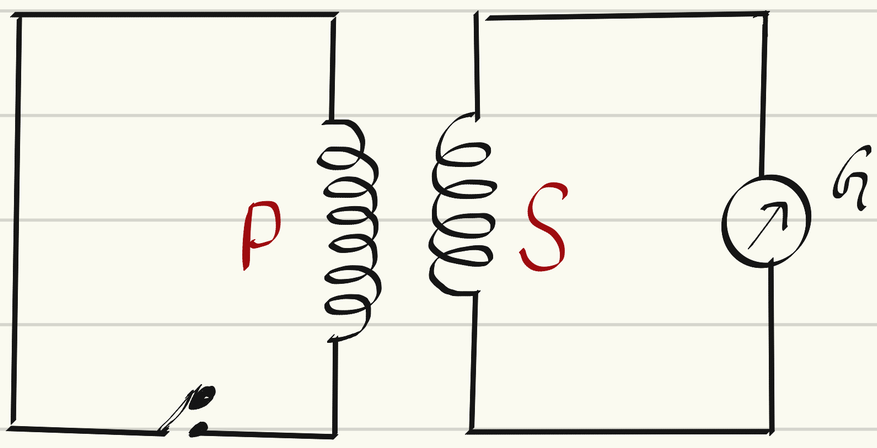
for a coil of N turns

$$N\phi \propto I$$

the constant of proportionality is called Inductance.

* It depends only on the geometry of the coil and intrinsic material properties. (same as in capacitance)
* It is scalar quantity
* Dimensional formula = $[ML^2T^{-2}A^{-2}]$ $\left[\text{by } \frac{\phi}{I}\right]$
* SI unit - henry

Mutual Induction: It is the phenomenon of inducing emf in a coil due to the change of current with time in a nearby coil.



Mutual Inductance(M): It is the magnetic flux linked with the secondary coil if unit current flows in the primary coil.

$$\phi_s \propto I_p$$

or

$$\phi_s = M I_p$$

$\phi_s \rightarrow$ flux linked with secondary coil

$I_p \rightarrow$ current in primary coil

If $I_p = 1$, $\boxed{M = \phi_s}$

According to Faraday's law

$$\mathcal{E}_s = -\frac{d\phi}{dt}$$

$\mathcal{E}_s \rightarrow$ induced emf in secondary coil

or

$$\mathcal{E}_s = -\frac{d(MI_p)}{dt}$$

$$\boxed{\mathcal{E}_s = -M\frac{dI_p}{dt}}$$

If $-\frac{dI_p}{dt} = 1$

$$\mathcal{E}_s = M$$

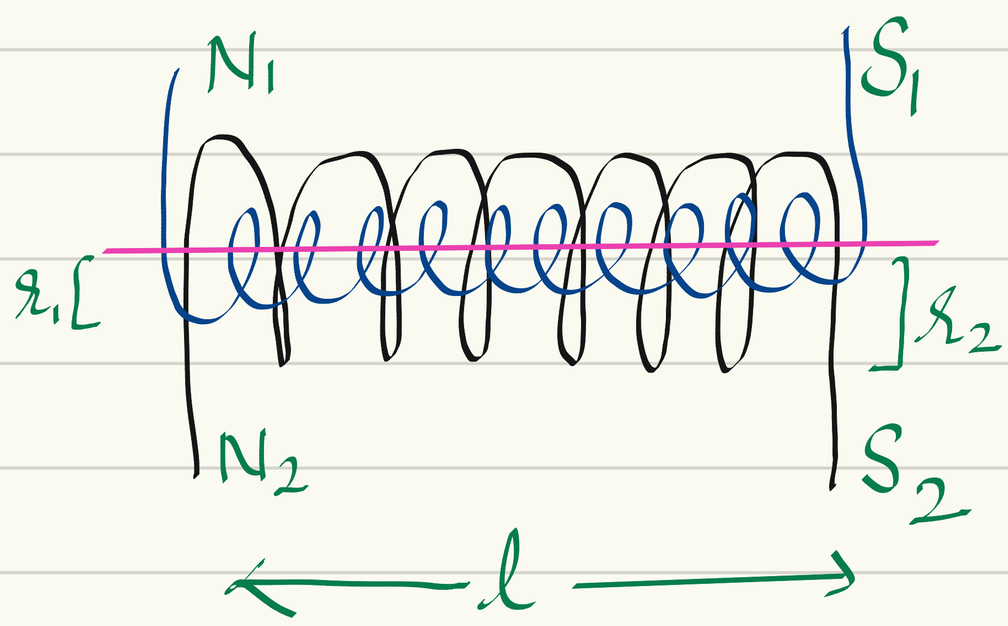
i.e

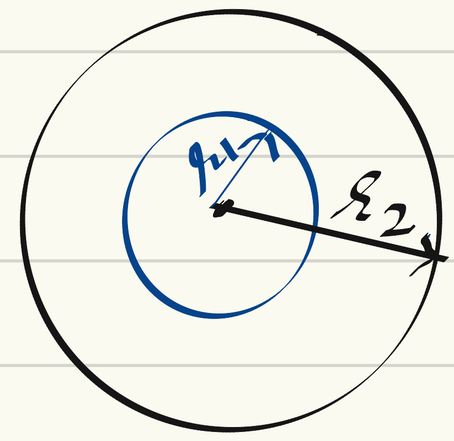
The mutual inductance of two coils can be defined as the induced emf produced in the secondary coil due to unit rate of decrease in current in primary coil.

SI unit - henry

Mutual Inductance of Two long Co-axial Solenoid:

Consider two coaxial solenoid S_1 and S_2 such that S_1 surrounded by S_2 as shown in fig. Length of the coils is l. N_1 and N_2 are no. of turns. r_1 and r_2 are radius.





$N_1 = n_1 l$
$N_2 = n_2 l$

n_1 and n_2 are no. of turns per unit length of coils S_1 and S_2 respectively.

Magnetic field inside the solenoid S_2 is

$$B_2 = \mu_0 n_2 I_2$$

Magnetic flux linked with coil S_1

$$N_1 \phi_1 = (n_1 l)(\mu_0 n_2 I_2)(\pi R_1^2) \qquad [\because N_1 = n_1 l, \ \phi_1 = B_2 A_1]$$

$$N_1 \phi_1 = \mu_0 \pi n_1 n_2 I_2 R_1^2 l$$

but $N_1 \phi_1 = M_{12} I_2$ [$\because \phi_1$ is due to I_2]

$$M_{12} I_2 = \mu_0 \pi n_1 n_2 I_2 R_1^2 l$$

or $$M_{12} = \mu_0 \pi n_1 n_2 R_1^2 l \quad \text{—(1)}$$

Now mag. field inside S_1

$$B_1 = \mu_0 n_1 I_1$$

flux linked with S_2

$$N_2 \phi_2 = (n_2 l)(\mu_0 n_1 I_1)(\pi R_1^2)$$

flux due to I_1 in S_1 is confined solely S_1 so

area $= \pi R_1^2$

put $N_2 \phi_2 = M_{21} I_1$

then

$$M_{21} I_1 = (n_2 l)(\mu_0 n_1 I_1)(\pi R_1^2)$$

or

$$M_{21} = \mu_0 \pi n_1 n_2 R_1^2 l \quad \text{—(2)}$$

from eq/n (1) and (2)

$$M_{12} = M_{21} = M \text{ (say)}$$

also $M = \mu_0 n_1 n_2 \pi R_1^2 l$

for a medium

$$M = \mu n_1 n_2 \pi R_1^2 l$$

$$\boxed{M = \mu_0 \mu_r n_1 n_2 \pi R_1^2 l}$$

* Mutual inductance of coils, solenoids etc depends on there separation and their relative orientation.

Self Induction :- Self induction is the property of the coil due to which it opposes the increase or decrease of current flowing through it.

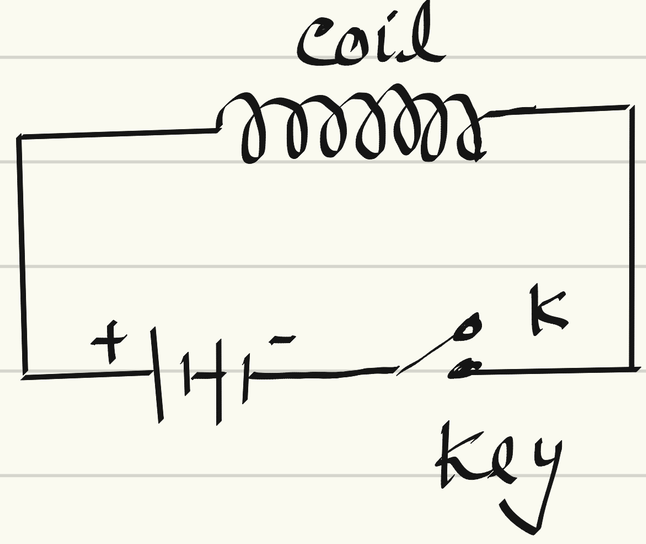
* It is also known as inertia of electricity.

Self Inductance (L) : The magnetic flux linked with the coil when unit current flows through it.

Let I current flows in a coil. Then magnetic flux

$$\phi \propto I$$

or $$\phi = LI$$



Where L is coefficient of self-induction or 'self inductance'.

If $I = 1$ A then

$$\phi = L$$

According Faraday's law

$$\mathcal{E} = -\frac{d\phi}{dt}$$

or $\mathcal{E} = -\frac{d}{dt}(LI)$ $\quad [\phi = LI]$

or $\boxed{\mathcal{E} = -L\frac{dI}{dt}}$

If $-\frac{dI}{dt} = 1$ then $L = \mathcal{E}$

Thus self inductance of a coil is numerically equal to the emf induced across the end of the coil when the rate of change of current is unity (1 A/s)

SI unit → henry , 1 henry = 1 $V A^{-1} s$

Inductor: A coil of insulated wire which opposes the change in current.

Symbol →

* An ideal inductor has high value of 'L' (self inductance) and zero ohmic resistance.

Self Inductance of Solenoid:

Let the magnetic field inside the solenoid

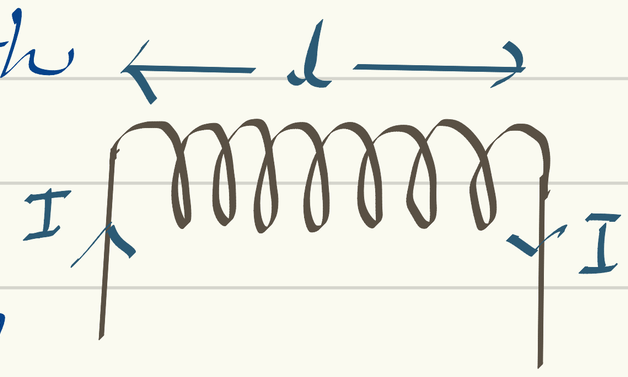
$B = \mu_0 n I$

n is no. of turns per unit length

Total no. of turns $N = nl$

Flux linked with each turn

$\phi = (\mu_0 n I) A \quad [\because \phi = BA]$



for N turns

$\phi = (nl)(\mu_0 n I) A \quad [\because nl = N]$

$\phi = \mu_0 n^2 I A l$

but $\phi = LI$, then

$LI = \mu_0 n^2 I A l$

Or $\boxed{L = \mu_0 n^2 A l}$

Or $L = \mu_0 \left(\frac{N}{l}\right)^2 A l$ $\left[n = \frac{N}{l}\right]$

Or $\boxed{L = \frac{\mu_0 N^2 A}{l}}$

Self Inductance of a solenoid depends on -

(I) $L \propto N$ (no. of turns)

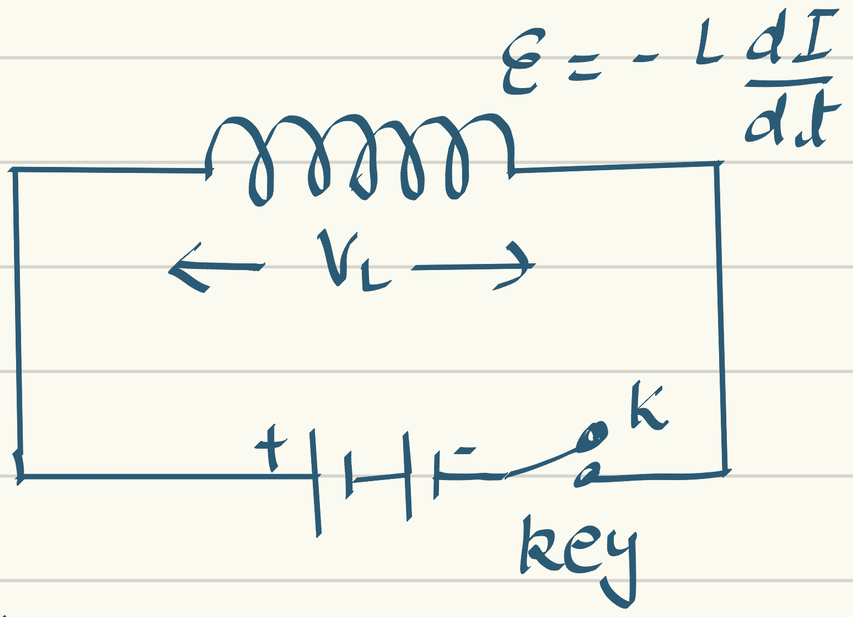
(II) $L \propto \frac{1}{l}$ (length of the coil)

(III) $L \propto A$ (Area of cross-section)

(IV) Medium insides the coil.

* In an iron rod is placed inside the coil (solenoid) it inductance increases.

Energy Stored In An Inductor:



When current flows through inductor an emf induces.

$$|\mathcal{E}| = L\frac{dI}{dt}$$

the rate of work done

$$\frac{dW}{dt} = |\mathcal{E}|\, I \qquad [\because dW = \mathcal{E} I\, dt]$$

$$\frac{dW}{dt} = LI\frac{dI}{dt}$$

$$W = \int dW$$

$$= \int L I \, dI = L \int I \, dI$$

$$= L \frac{I^2}{2}$$

$$W = \frac{1}{2} L I^2$$

or energy stored

$$\boxed{U_B = \frac{1}{2} L I^2}$$

Energy Density (u_B)

We have

$$U_B = \frac{1}{2} L I^2$$

$$= \frac{1}{2} L \left(\frac{B}{\mu_0 n}\right)^2 \quad \left[\because B = \mu_0 n I\right]$$

$$= \frac{1}{2} (\mu_0 n^2 A l) \left(\frac{B}{\mu_0 n}\right)^2 \quad [\because L = \mu_0 n^2 A l$$

$$U_B = \frac{1}{2\mu_0} B^2 A l$$

Mag. energy per unit volume
(mag energy density u_B)

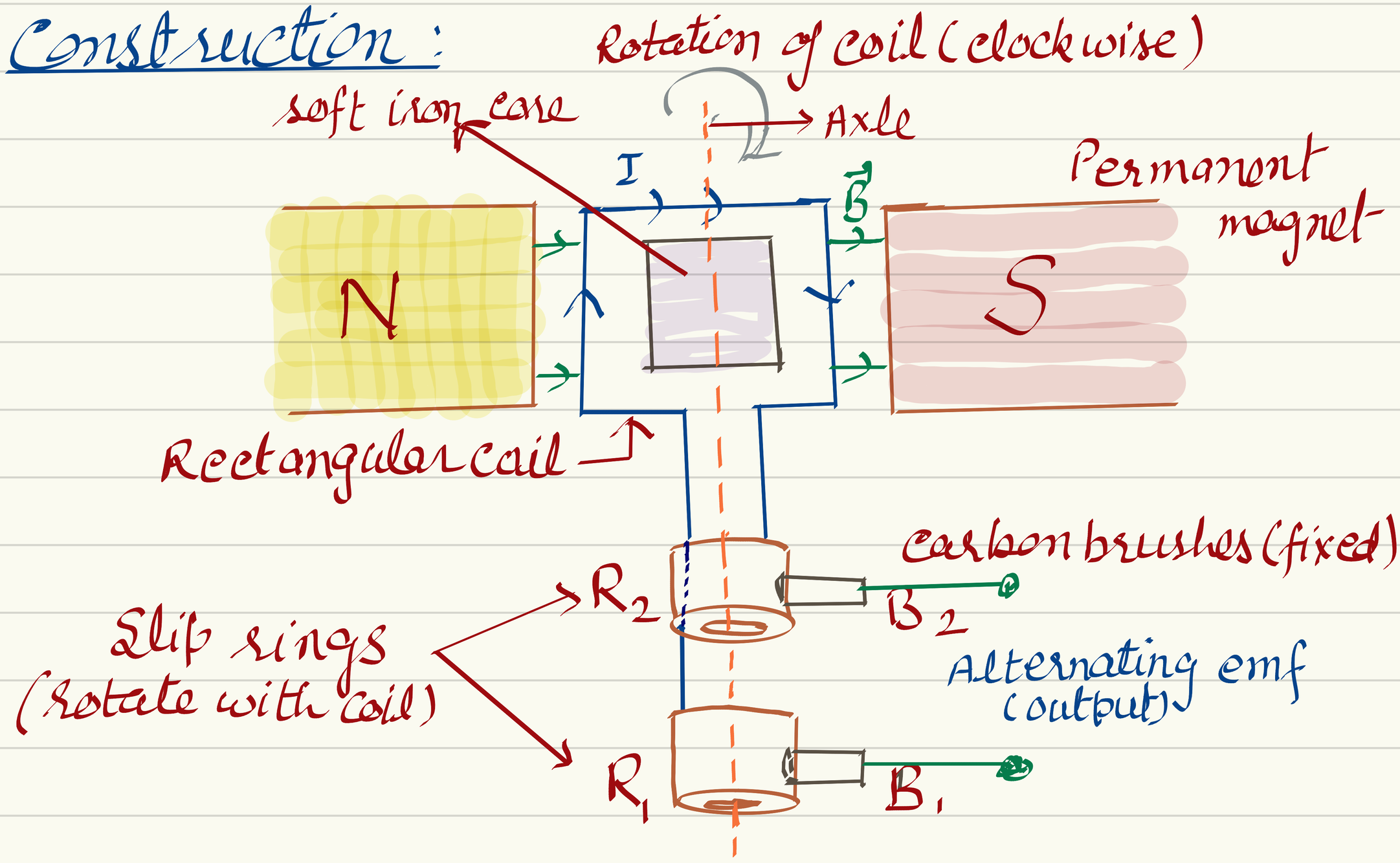
$$u_B = \frac{U_B}{Al} = \frac{1}{2\mu_0} \frac{B^2 A l}{A l}$$

or $$\boxed{u_B = \frac{B^2}{2\mu_0}}$$

AC Generator: An electrical machine used to convert mechanical energy into electrical energy is known as AC generator.

Principle: It works on the priniple of electromagnetic induction. ie. when a coil is rotated in uniform magnetic field an induced emf is produced in it.

Construction:



1. **Armature:** The rotating part of the AC generator is called an armature. It includes the commutator and coils.
2. **Commutator:** The contact where current leaves the brush to enter the armature windings.
3. **Brushes:** Two carbon brushes B_1 and B_2 are pressed against slip rings. The brushes remain fixed and output is obtained.
4. **Slip Rings:** The two ends of armature coil

are connected to two brass slip rings R_1 and R_2. Slip ring transmit electric power into a coil to make it rotating.

Streng magnetic field: A streng magnetic field is used in which coil rotates.

Working: When the coil rotates the magnetic flux changes due to which induced emf is generated and hence induced current flows in the coil.

Theory:

$$\phi = NBA\cos\theta \qquad [N \rightarrow \text{no. of turns in coil}]$$

or $$\phi = NBA\cos\omega t \qquad [\theta = \omega t]$$

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\omega t)$$

or $$\mathcal{E} = -NBA(-\sin\omega t)\omega \qquad \left[\frac{d}{dt}\cos\omega t = -\omega\sin\omega t\right]$$

$$\mathcal{E} = \omega NBA\sin\omega t \quad -(1)$$

for $\mathcal{E}_{max}$, $\sin\omega t = 1$, then

$$\mathcal{E}_0 = NBA\omega$$

or $$\mathcal{E}_0 = NBA\omega \qquad [\mathcal{E}_0 \rightarrow \text{Maximum emf}]$$

where $\mathcal{E}_0$ is maximum value of emf.

from eq^n (1)

$$\boxed{\mathcal{E} = \mathcal{E}_0\sin\omega t}$$

Instantaneous current

$$I = \frac{\mathcal{E}}{R}$$

or $I = \frac{\mathcal{E}_0}{R} \sin \omega t$

$$\boxed{I = I_0 \sin \omega t} \qquad \left[\frac{\mathcal{E}_0}{R} = I\right]$$

Variation in Induced emf:-

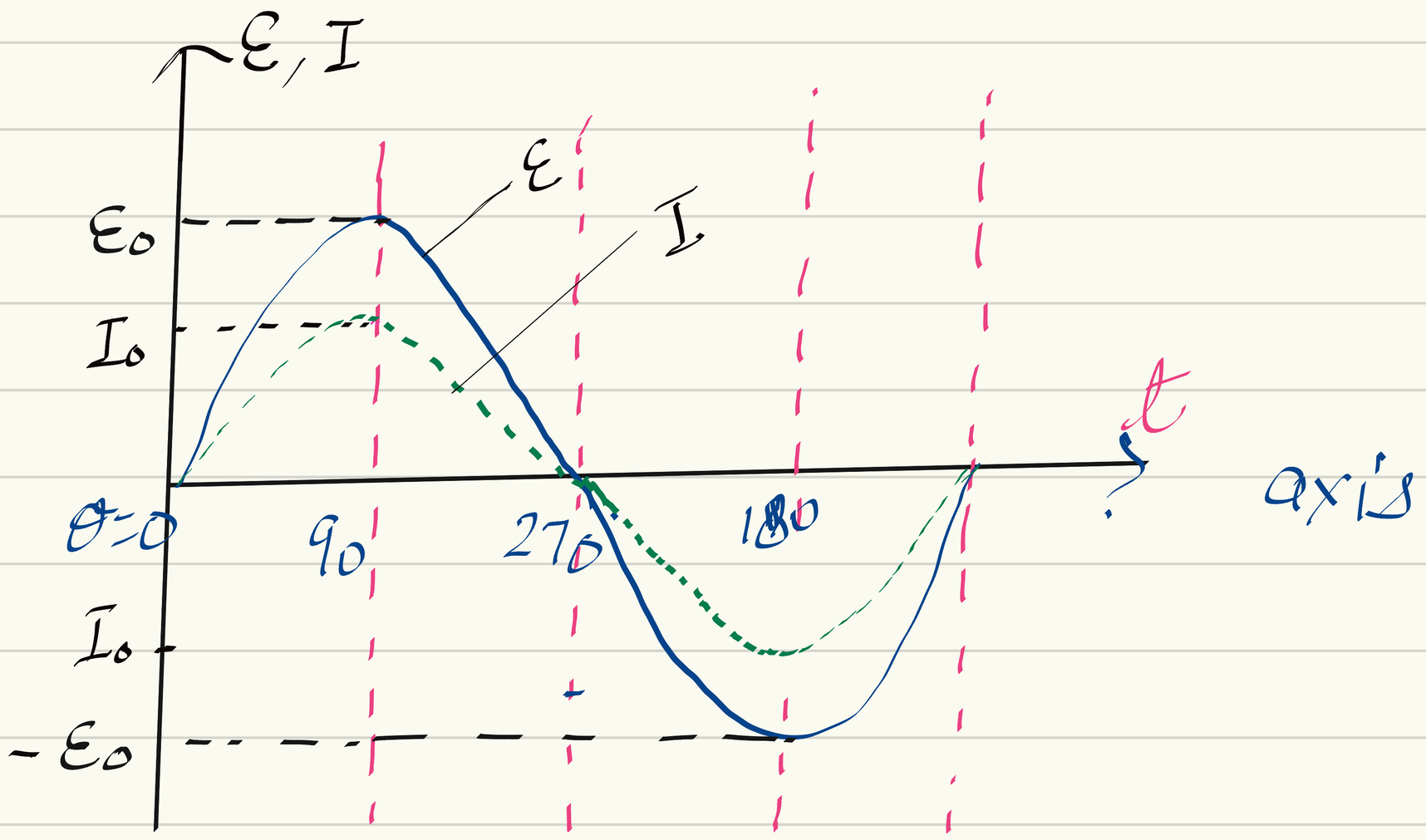
(I) When $\theta = 0 \Rightarrow \mathcal{E} = 0$

(II) When $\theta = 90^\circ$, $\mathcal{E} = \mathcal{E}_0$

(III) When $\theta = 180^\circ$, $\mathcal{E} = 0$

(IV) When $\theta = 270^\circ \Rightarrow \mathcal{E}_0 = -\mathcal{E}_0$

(V) When $\theta = 360^\circ \Rightarrow \mathcal{E}_0 = 0$



Unit IV: Electromagnetic Induction and Alternating Currents 24 Periods

Chapter–6: Electromagnetic Induction

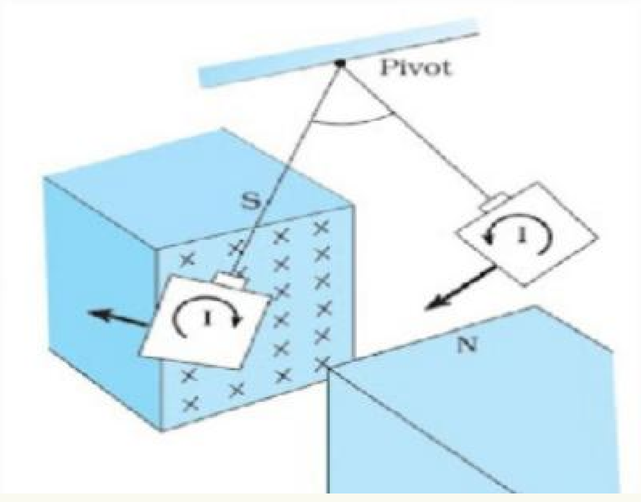
Electromagnetic induction; Faraday's laws, induced EMF and current; Lenz's Law, Self and mutual induction.

Not in CBSE Syllabus

Eddy Currents

When bulk pieces of conductors are subjected to changing magnetic flux, induced currents are produced in them, these currents are called **eddy currents**.

Eddy currents are undesirable since they heat up the core and dissipate electrical energy in the form of heat. **Eddy currents** are minimized by using laminations of metal to make a metal core. This arrangement reduces the strength of the eddy currents.



Eddy currents are used to advantage in certain applications like:

- **Magnetic braking in trains:** Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train.

- **Electromagnetic damping:** Certain galvanometers have a fixed core made of non-magnetic metallic material. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.

- **Induction furnace:** Induction furnace can be used to produce high temperatures. A high-frequency alternating current is passed through a coil that surrounds the metals to be melted. The eddy currents generated in the metals produce high temperatures sufficient to melt them.

- **Electric power meters:** The shiny metal disc in the electric power meter rotates due to

the eddy currents. Electric currents are induced in the disc by magnetic fields produced by sinusoidally varying currents in a coil.