* Amperian loop is an imaginary closed loop around the conductor.

Ampere's Circuital Law:

This law states the relationship between the current and the magnetic field created by it.

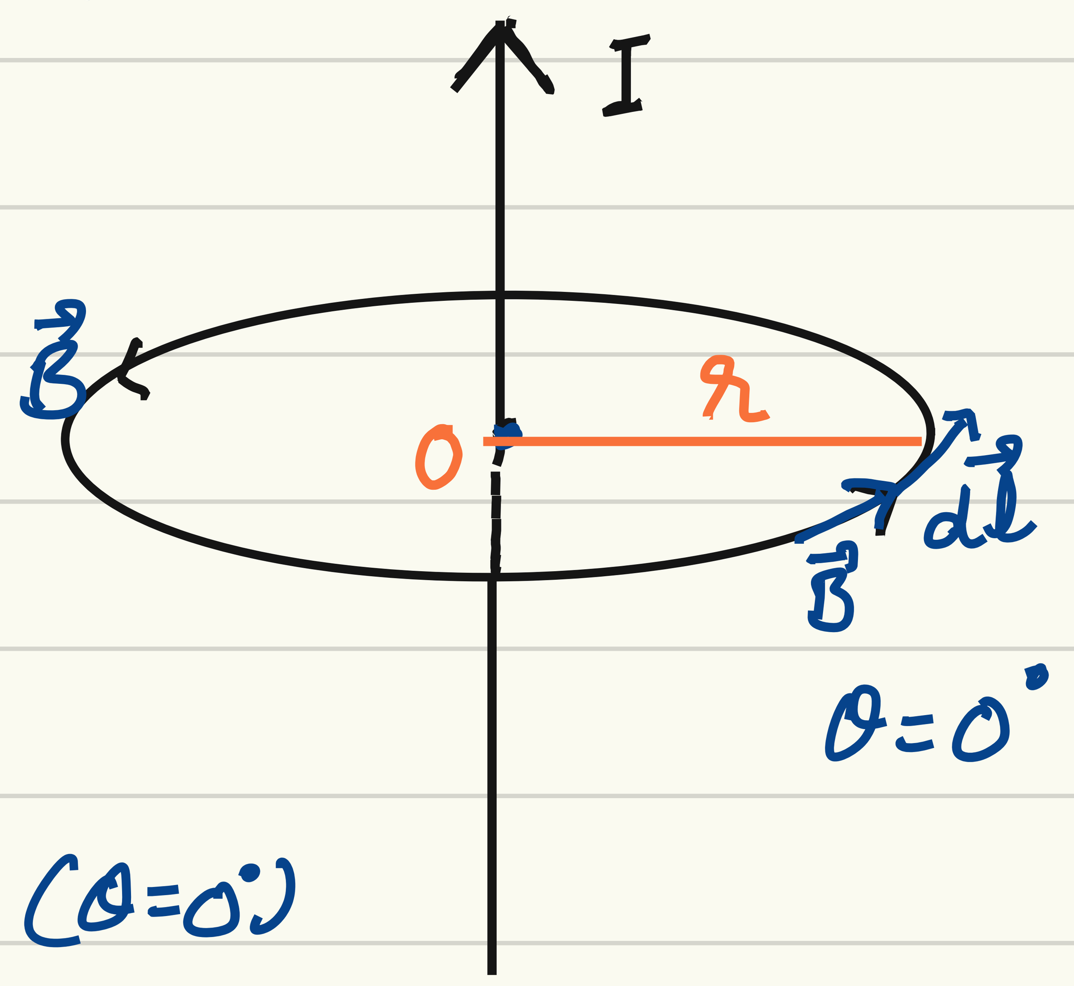
"According to this law the integral of magnetic field density (B) along an imaginary closed path is equal to the μ_0 times of the current enclosed by the path."

OR

"The line integral of the magnetic field ($\vec{B}$) around any closed path is equal to μ_0 times the total current (i) passing through the closed path."

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

where μ_0 is the permeability of free space.



Proof: From Biot-Savart's law,
Magnetic field due to long straight wire

$$B = \frac{\mu_0 I}{2\pi r}$$

here $\vec{B}$ and $d\vec{l}$ are in same direction ($\theta = 0°$)

So

$$\oint \vec{B}\cdot d\vec{l} = \oint B\,dl \cos 0° = \oint B\,dl \qquad [\cos 0° = 1]$$

and

$$= B \oint dl$$

$$= \frac{\mu_0 I}{2\pi r}\oint dl \qquad \left[\because B = \frac{\mu_0 I}{2\pi r}\right]$$

$$= \frac{\mu_0 I}{2\pi r} \times 2\pi r \qquad \left[\because \oint dl = 2\pi r\right]$$

or

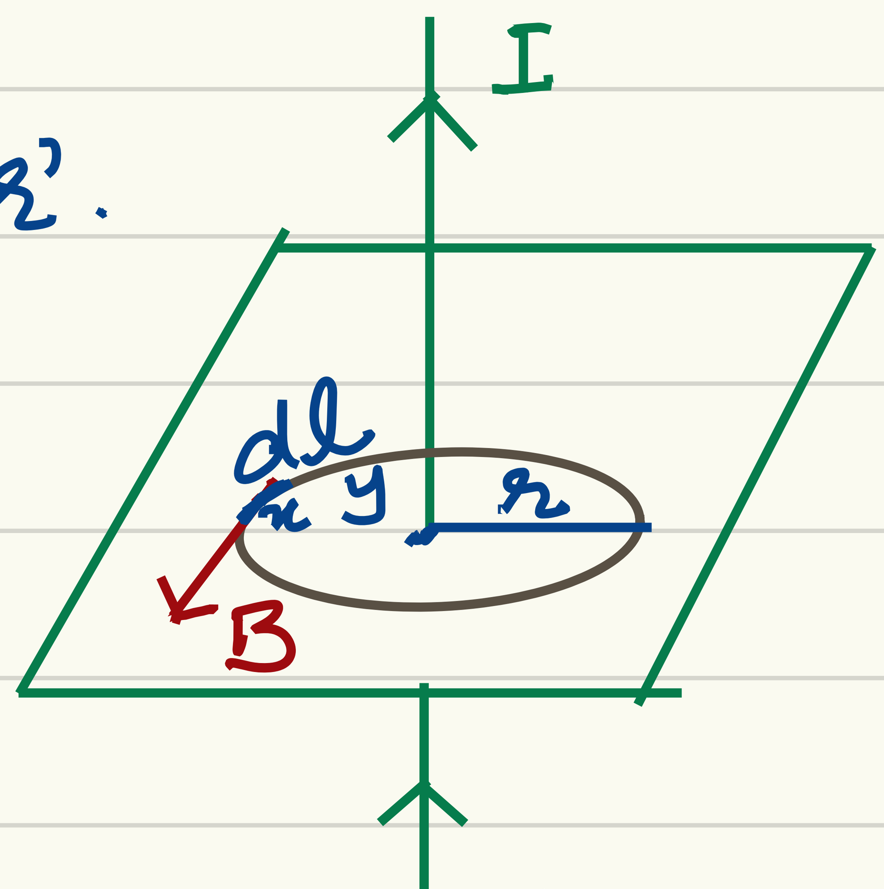
$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I}$$

* This relation involves a sign convention given by right hand thumb rule.

* Ampere's law is to Biot-Savart law what Gauss's law is to coulomb's law.
* It is possible to choose the amperian loop such that at each point of the loop, either
  (i) B is tangential to the loop (B is constant and non zero)
  (ii) B is normal to the loop, or
  (iii) B vanishes.
* Bio Savart law based on the experimental results whereas Ampere's law based on mathematical

Applications of Ampere's Circuital law

1. Magnetic field due to thin current carrying straight conductor



Consider a radius of radius 'r'.
Let xy be the small element of length dl. $\vec{B}$ and $\vec{dl}$ are in the same dir^n ($\theta = 0°$)

By A.C.L.

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I \qquad I \rightarrow \text{enclosed}$$

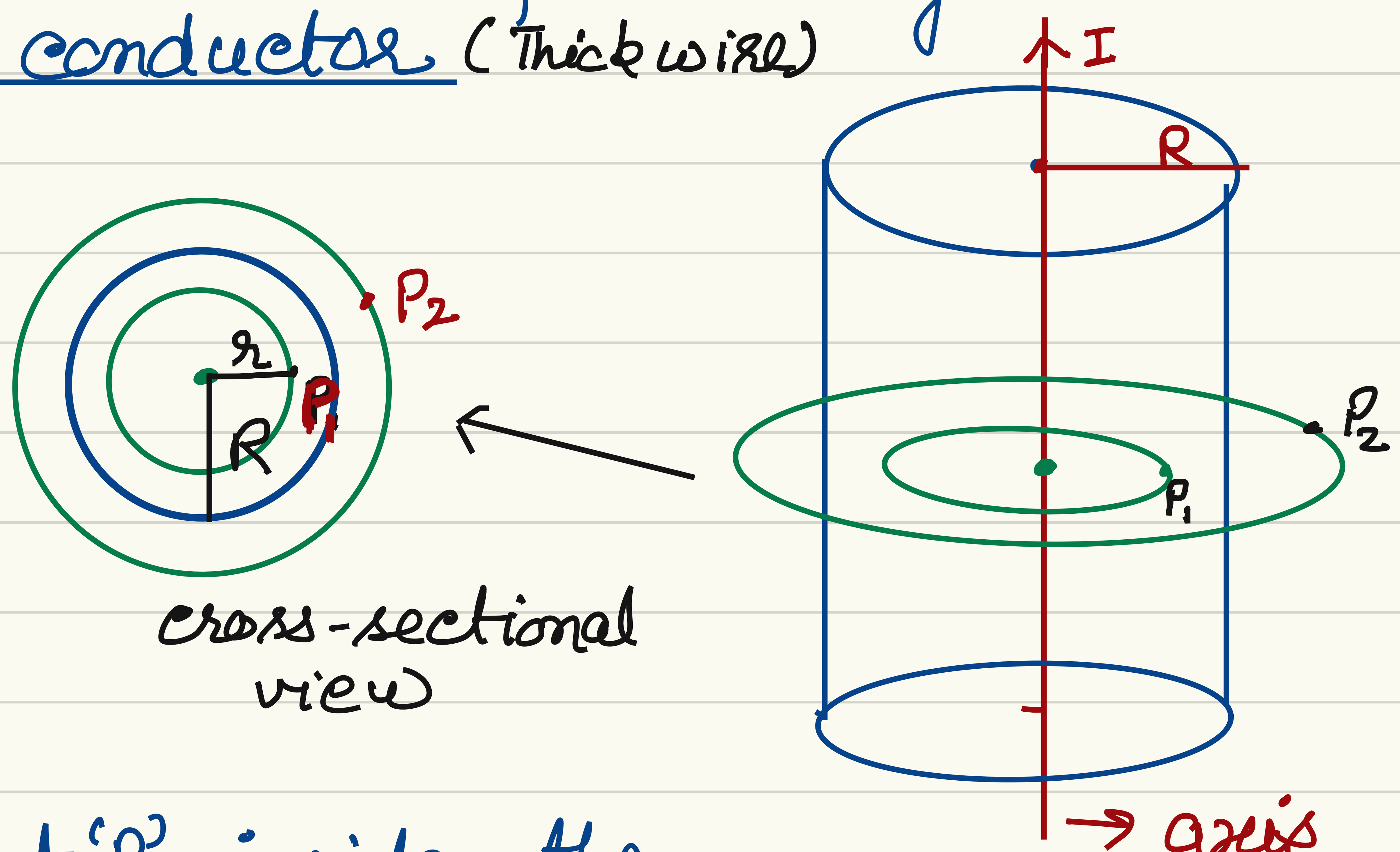
$$\oint B\,dl\cos 0° = \mu_0 I$$

$$B\oint dl = \mu_0 I$$

$$B \times 2\pi r = \mu_0 I \qquad \left[\because \oint dl = 2\pi r\right]$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

2 Magnetic field due to infinite long solid cylindrical conductor (Thick wire)



(i) For a point 'P_1' inside the cylinder, ($r < R$)

current from area $\pi R^2 = I$

so current from area πr^2 (I_{en}) $= \frac{I}{\pi R^2} \times \pi r^2$

$$I_{en} = \frac{I r^2}{R^2}$$

By ACL for 'P_1'

$B \times 2\pi r = \mu_0 I_{en}$ $\left\{ \because \oint \vec{B} \cdot \vec{dl} = B \times 2\pi r \right.$

or $B \cdot 2\pi \cancel{r} = \mu_0 \frac{I r^{\cancel{2}}}{R^2}$

or $\boxed{B = \frac{\mu_0 I r}{2\pi R^2}} \Rightarrow \boxed{B \propto r}$

(ii) For a point on the axis of cylinder

as $r = 0$, $B = 0$

i.e $B_{axis} = 0$

(iii) for a point on the surface of cylinder
(r = R)

$$B_S \times 2\pi R = \mu_0 I$$

or
$$B_S = \frac{\mu_0 I}{2\pi R} \quad [B_{max}]$$

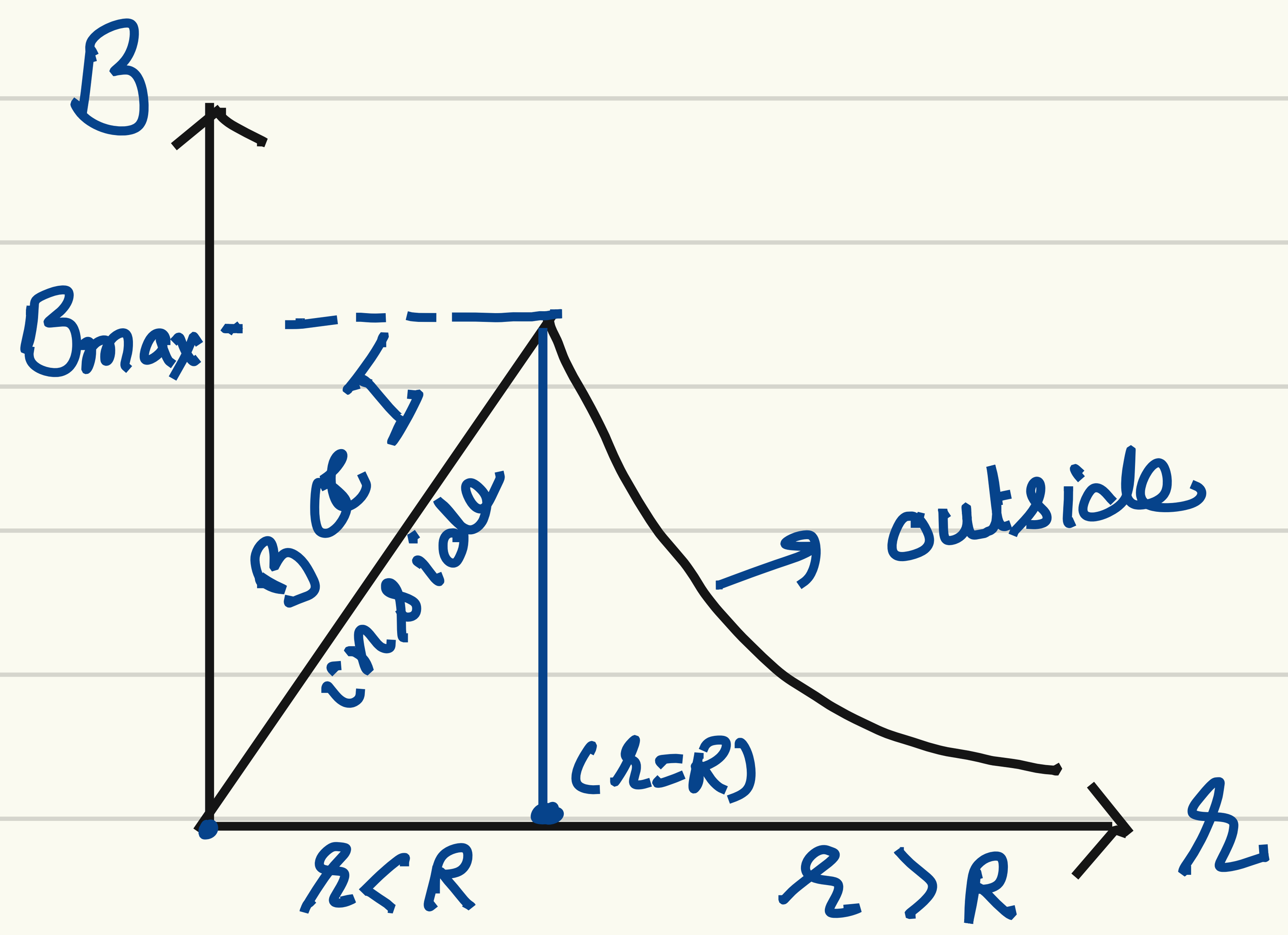
(iv) for a point outside the cylinder
(r > R)

$$B_{out} \times 2\pi r = \mu_0 I$$

or
$$B_{out} = \frac{\mu_0 I}{2\pi r} \Rightarrow \boxed{B \propto \frac{1}{r}}$$

* magnetic field outside the cylinder conductor (wire) does not depend upon nature of conductor like radius, area, solid, hollow etc.

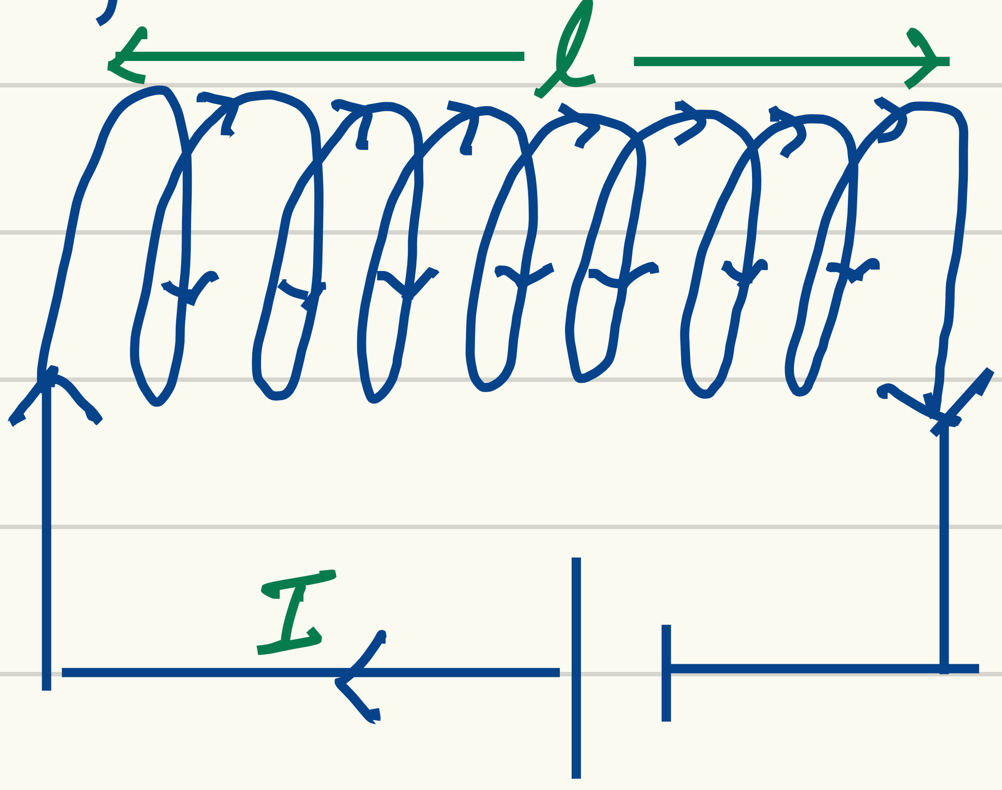
* Graph b/w B and r

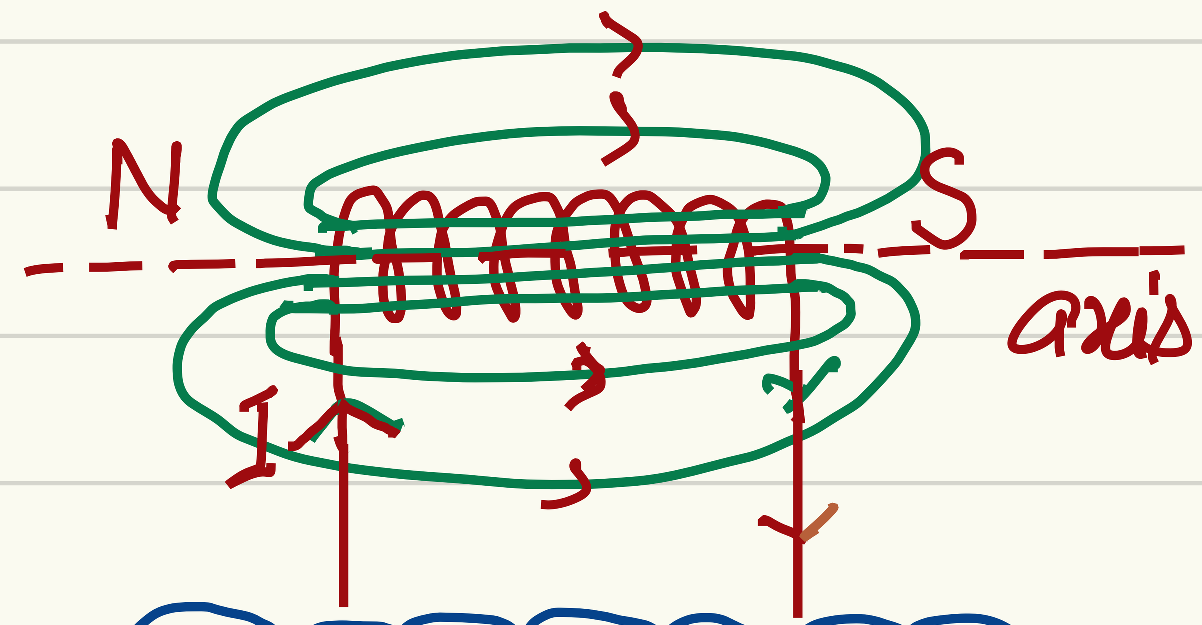


Solenoid

It is a coil which has length and used to produced magnetic field of long range.
It consists a conducting wire tightly wound over a cylindrical frame in the form of helix.

Magnetic field due to a long solenoid





Solenoid behaves like a bar magnet

Formula

$$B = \mu_0 n I$$

$$= \mu_0 \frac{N}{l} I$$

$n \rightarrow$ No. of turns per unit length
$N \rightarrow$ Total turns
$l \rightarrow$ Length of solenoid

* Magnetic field of a solenoid can be increased by inserting an iron rod.

* Magnetic field at both axial end points of solenoid is half.

$$B = \frac{1}{2} \mu_0 n I$$

Magnetic field due to Toroid

mag. field produces within the toroid

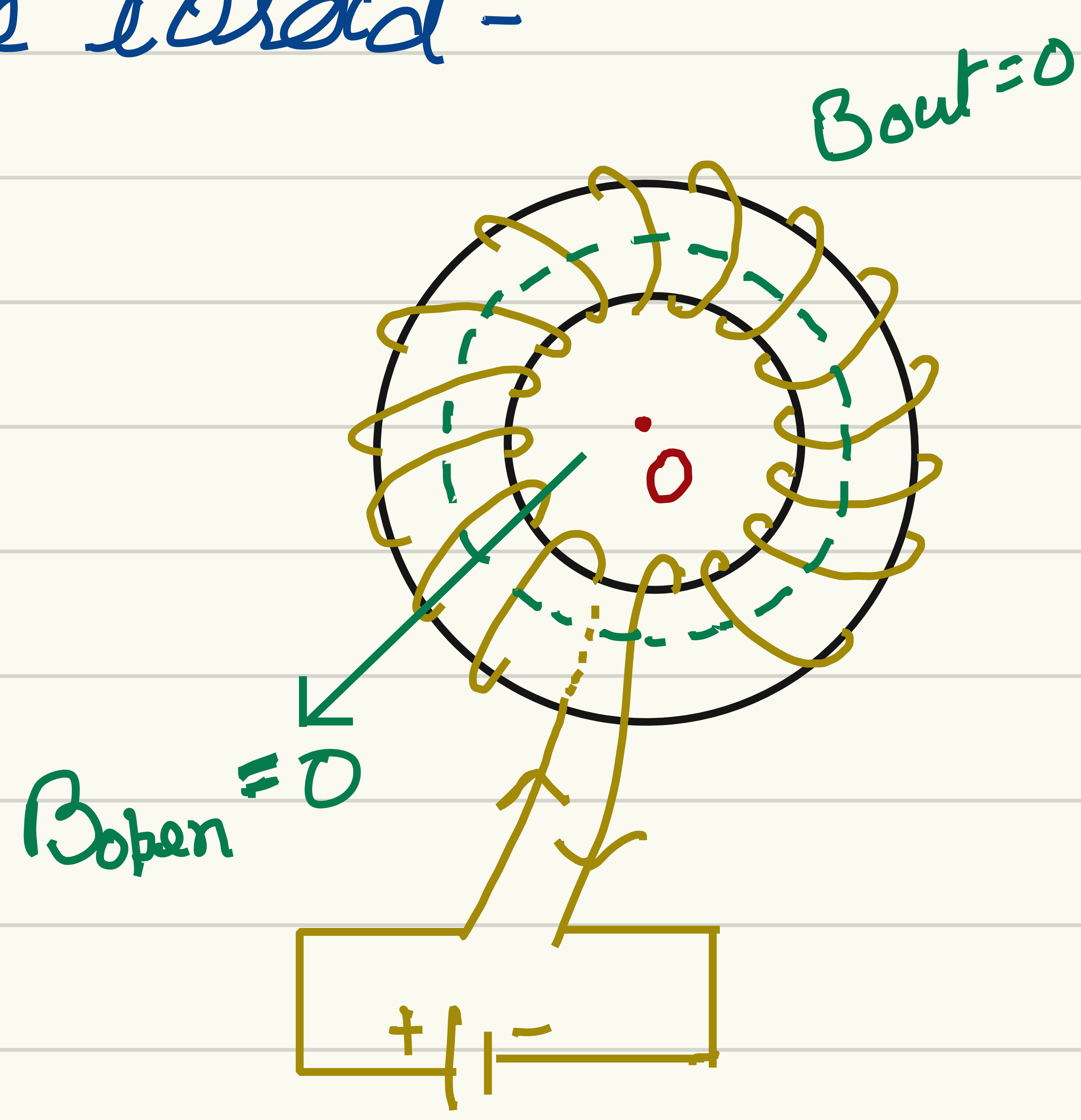
A toroid can be considered as a ring shaped closed solenoid also called endless solenoid.

Magnetic field inside the toroid -

$$B = \mu_0 n I$$

where

$$n = \frac{N}{2\pi R}$$



* Loop encloses no current.
ie $I_{en} = 0 \Rightarrow B_{open} = 0$

* $B_{outside} = 0$, since
* In an ideal toroid the coils are circular. In reality the turns form helix and there is always a small magnetic field external to the toroid

Uses of solenoid and toroid:

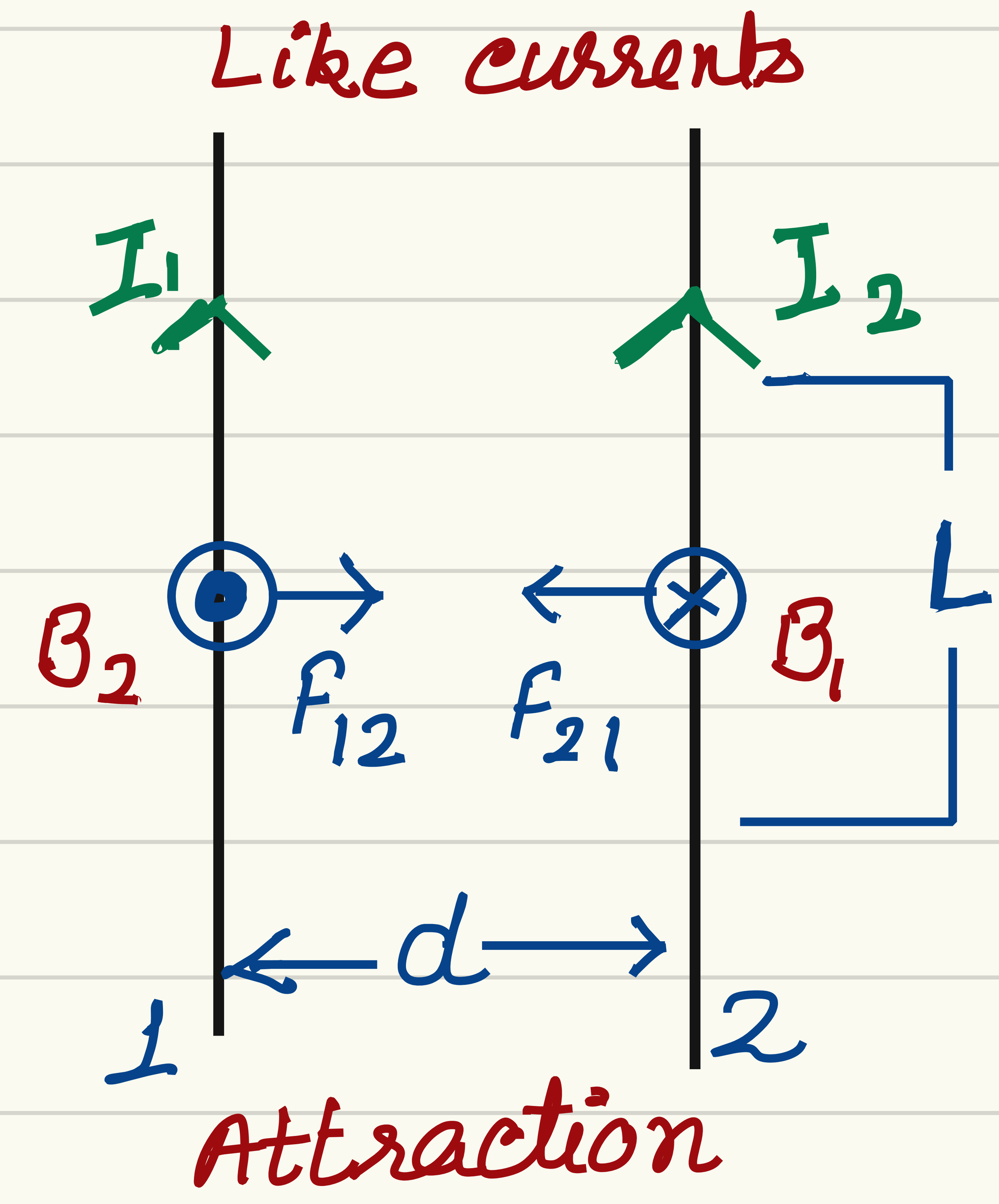
- In television to generate magnetic field.
- Synchrotron uses both, solenoid and toroid to generate high magnetic field.

Difference b/w Solenoid and electromagnet

Solenoid is just a coil of wire but when current flows through the coil, it is called an electromagnet.

Force Between Two Parallel Currents:

Fig. shows two long parallel conductors 1 & 2 separated by a distance d and carrying (parallel) currents I_1 and I_2 respectively. Let I_1 produces mag. field B_1 and I_2 produces B_2 mag field.



$L \rightarrow$ wire's length segment

Force on wire 1 due the field of wire 2 (B_2)

$$F_{12} = I_1 L B_2 \qquad [\theta = 90°]$$

$$= I L \left(\frac{\mu_0 I_2}{2\pi d} \right)$$

$$= \frac{\mu_0 I_1 I_2}{2\pi d} L$$

$$F_{12} = \frac{\mu_0}{4\pi} \frac{2 I_1 I_2}{d} . L$$

Similarly.

$$F_{21} = I_2 L B_1$$
$$= I_2 L \left(\frac{\mu_0 I_1}{2\pi d} \right)$$
$$= \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{d} \cdot L$$

$$F_{21} = \frac{\mu_0}{4\pi} \frac{2 I_1 I_2}{d} \cdot L$$

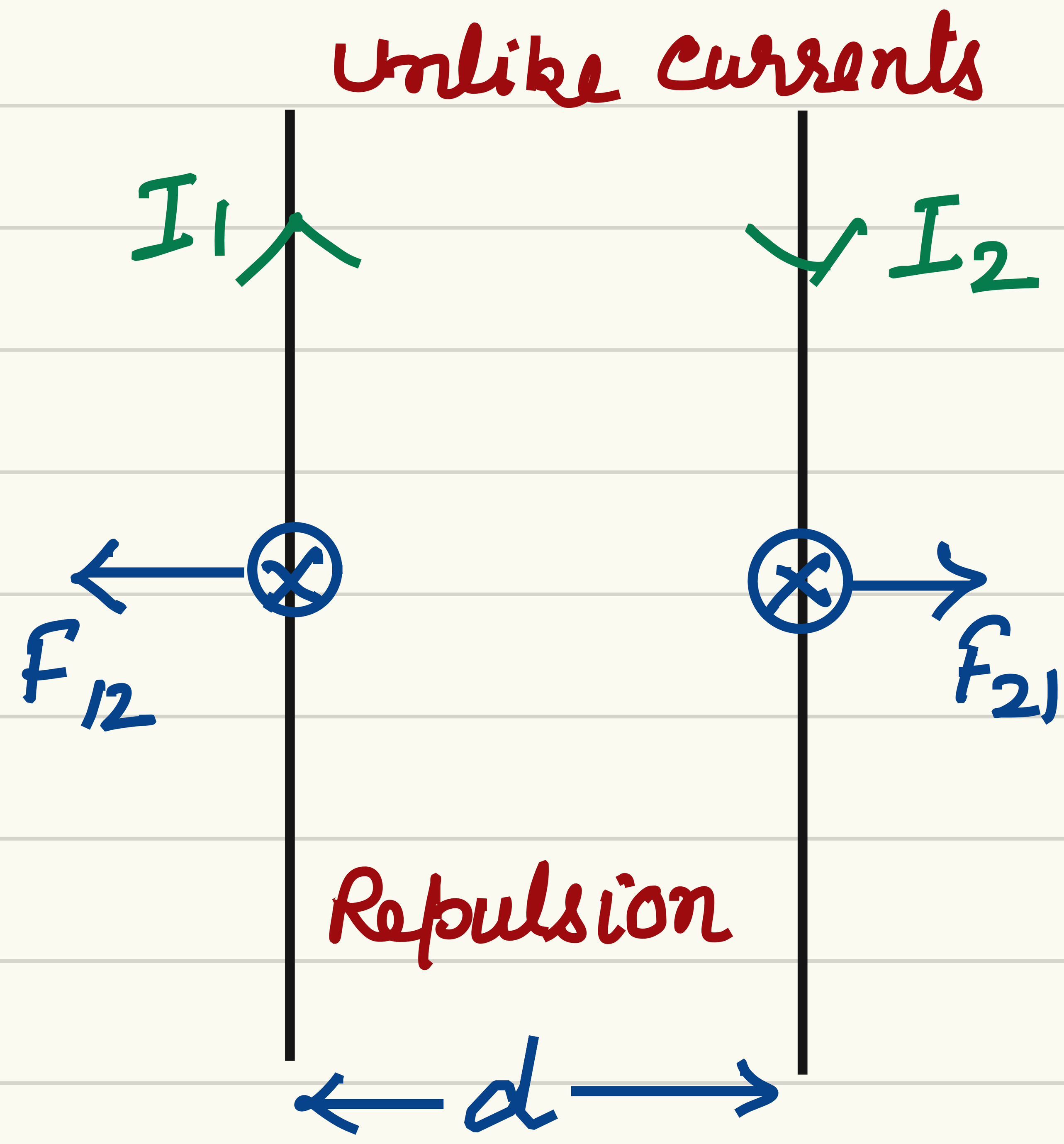
force per unit length

$$\frac{F_{12}}{L} = \frac{F_{21}}{L} = \frac{\mu_0}{4\pi} \frac{2 I_1 I_2}{d}$$

Forces $\vec{F}_{12}$ and $\vec{F}_{21}$ are equal in magnitude but opposit in dirn.

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

This verifies Newton's III Law.



*Parallel currents → Attract

Antiparallel currents → Repel

* Force per unit length is used to define the Ampere (A)

One Ampere: Ampere is the current which passes through each of two parallel infinite long straight conductor paced in free space at a distance of 1 m produces a force of 2×10^{-7} N/m.

$$f = \frac{F_{12}}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

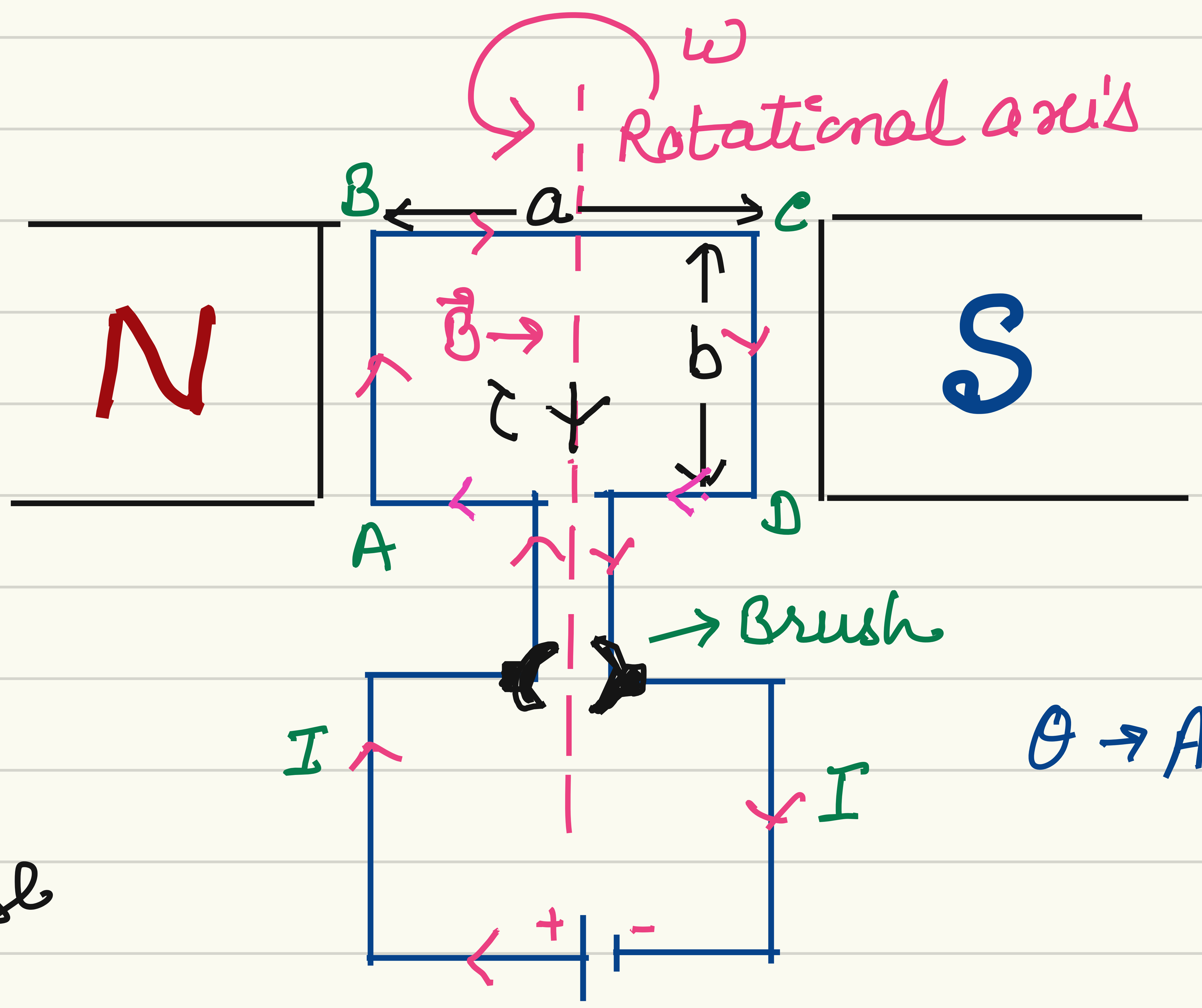
$$f = \frac{2\times10^{-7}(1)\times(1)}{1}$$

or $$f = 2\times10^{-7} \text{ N/m}$$

[* An instrument called current balance is used to measure this mechanical force]

Torque On Current Loop, Magnetic Dipole

Fig shows a rectangular loop carrying a steady current I, placed in uniform mag. field $\vec{B}$ and experiences a torque.



$\theta \rightarrow$ Angle b/w $\vec{B}$ and $\vec{A}$
Area vector

General case

Case I When mag. field is in the plane of the loop. $(\theta = 90°)$

$\vec{F}_{AB} = -\vec{F}_{CD}$ [by F.L.H. Rule]

i.e. $F_{net} = 0$

and $F_{AD} = F_{BC} = 0$ [|| and anti || to $\vec{B}$]

There will a torque on the loop as $F_{net} = 0$

We know $\tau = |Force| \times \perp$ distance

$$= F_1 \times \frac{a}{2} + F_2 \times \frac{a}{2}$$

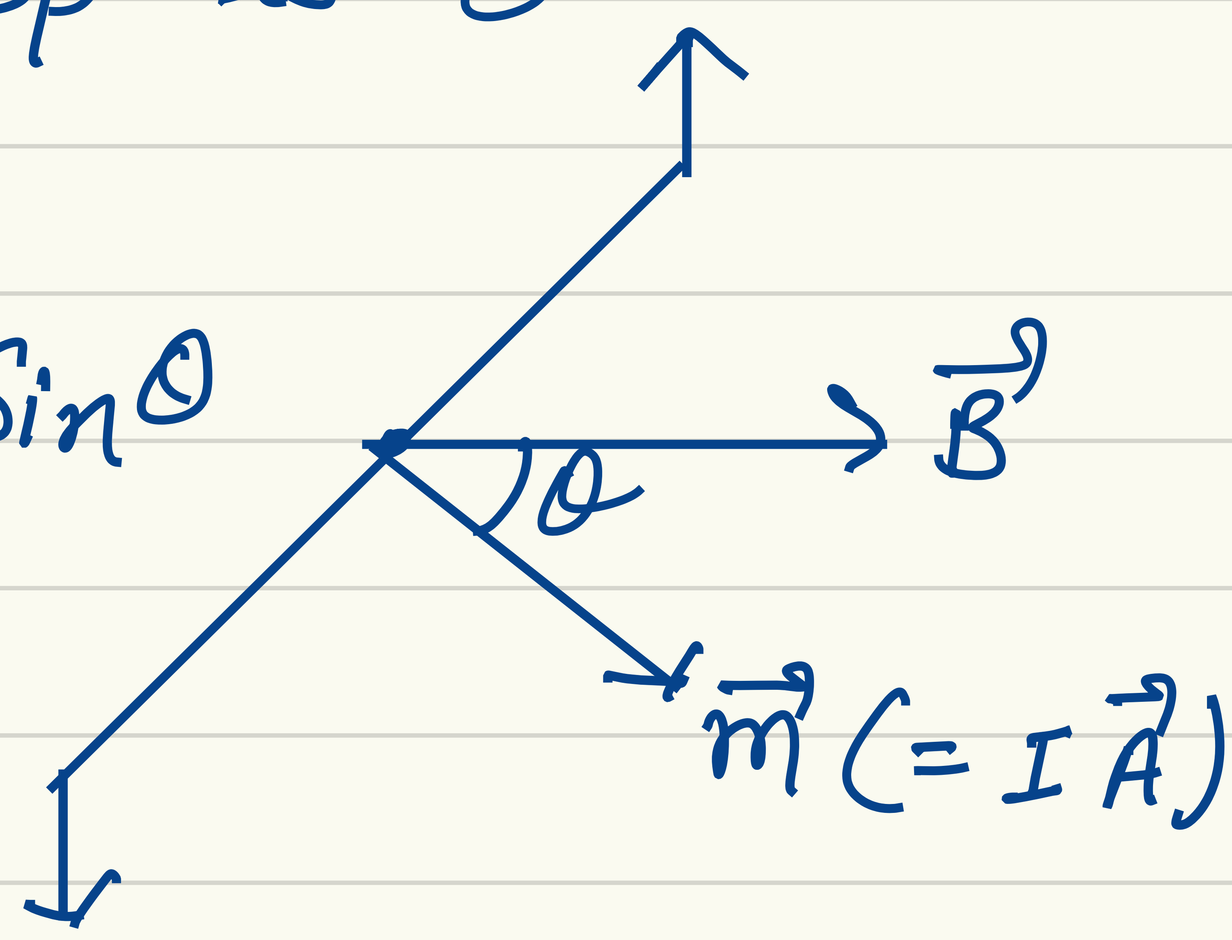
$$= IbB \cdot \frac{a}{2} + IbB \cdot \frac{a}{2} \quad \left[\begin{matrix} F_1 = F_2 = IbB \\ l \to b \end{matrix}\right]$$

$$= I(ab)B$$

$$\boxed{\tau = IAB} \qquad [ab = \text{Area } A]$$

<u>Case II</u>: When mag. field is not in the place of the loop

Let angle b/w mag. field B and normal to the plane of loop is θ



here,

$$\tau = F_1 \frac{a}{2} \sin\theta + F_2 \frac{a}{2} \sin\theta$$

$$= I(ab) B \sin\theta$$

$$\boxed{\tau = IAB\sin\theta}$$

for N turns

$$\boxed{\tau = NIAB\sin\theta}$$

vector form, $\tau = NI(\underbrace{\vec{A}}_{\text{area vector}} \times \underbrace{\vec{B}}_{\text{mag. field}})\sin\theta$

here $I\vec{A} = \vec{m} \to$ magnetic moment

* dir^n of $\vec{m}$ is same to $\vec{A}$.

Magnetic Moment (m) - It is the product of current I and area vector $\vec{A}$.

$$\vec{m} = I\vec{A}$$

$\vec{m}$ is a vector in the dir^n of $\vec{A}$.
S.I unit - Am^2

Now by $\tau = NIAB \sin\theta$

Case(I) If $\theta = 0°$
$\Rightarrow \tau = 0$ [Equilibrium position]

(II) If $\theta = 90°$
$\Rightarrow \tau = NIAB$ [Max. torque τ]

When $\theta = 90°$ τ is max.
This is called radial field ($\theta = 90°$)

Torque in terms of magnetic moment

$$\tau = NIAB \sin\theta$$

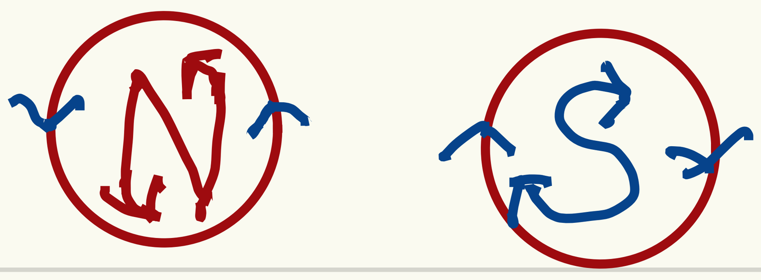
$$\tau = mB\sin\theta \qquad [m = NIA]$$

$$\boxed{\vec{\tau} = \vec{m} \times \vec{B}}$$

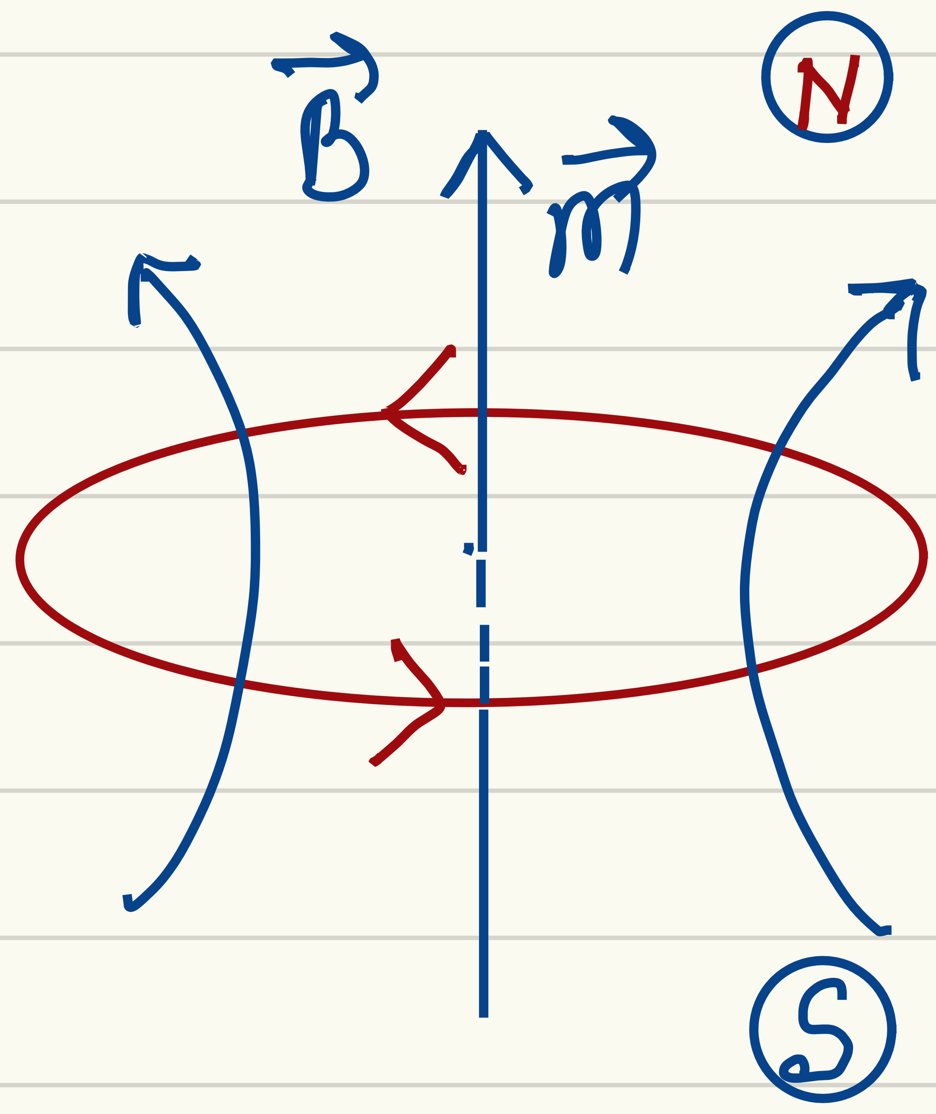
Dir^n of torque is $\perp$ to the plane of $\vec{m} \times \vec{B}$ and given by right hand thumb rule.

Circular Current Loop as a Magnetic Dipole

Magnetic field on the axis of a circular loop, of radius R, carrying current I is given by -



$$B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$



For $x >> R$

$$B = \frac{\mu_0 I R^2}{2 x^3}$$

$$= \frac{\mu_0 I (\pi R^2)}{2\pi x^3}$$

$$= \frac{\mu_0 I A}{2\pi x^3} \qquad [A = \pi R^2]$$

$$\boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{2m}{x^3}} \qquad [\because m = IA]$$

* This expression is similar to $\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{2p}{x^3}}$

$\mu_0 \to \frac{1}{\epsilon_0}$ (permittivity), $m \to p$ (electric dipole moment) and $B \to E$ (electric field)

* B for a point in the plane of the loop at a distance x from the centre –

$$B = \frac{\mu_0}{4\pi} \frac{m}{x^3} \qquad [x >> R]$$

* Magnetic monopoles are not known to exist.

Important Results

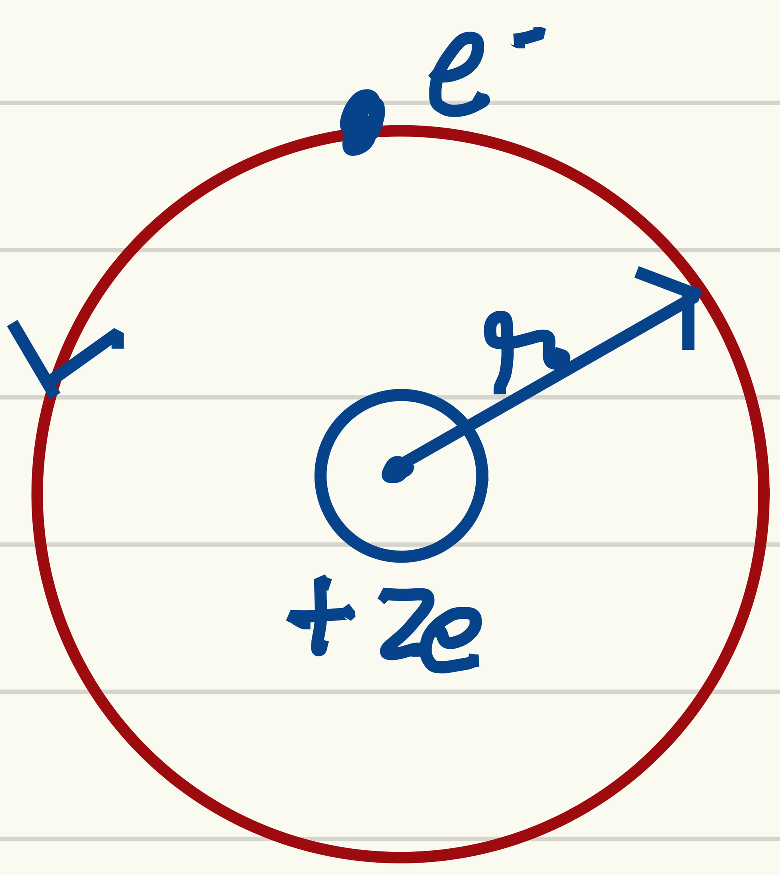
(1) Current loop produces a magnetic field and behaves like a magnetic dipole at large distances.

(ii) current loop is subjected to torque like a magnetic needle.

(iii) Elementry particles such as electron, proton also carry intrinsic magnetic moment.

The Magnetic Dipole Moment of a Revolving Electron:

In fig. Bohr picture of electron is shown.



Current due to the circular motion of electron,

$$I = \frac{e}{T} \qquad e = 1.6 \times 10^{-19}\,C$$

here $\qquad T \to$ Time period

$$T = \frac{2\pi r}{v}$$

⊗ μ_l ↓ into the page

so $\qquad I = \frac{ev}{2\pi r}$

The magnetic moment associated with electron's current

$$\mu_l = I\pi r^2$$

[dirⁿ of μ_l is into the page]

$$= \left(\frac{ev}{2\cancel{\pi}\cancel{r}}\right) \times \cancel{\pi} r^{\cancel{2}}$$

$$\mu_l = \frac{evr}{2}$$

or $\qquad \mu_l = \frac{e}{2m_e}(m_e v r)$

$$= \frac{e}{2m_e} \cdot l$$

[$l \to$ angular momentum, $l = mvr$]

Vector form $\qquad \boxed{\vec{\mu_l} = -\frac{e}{2m_e}\vec{l}}$

-ve sign shows the opposite dir^n of μ_l and l.
for +ve charge μ_l and l are in same dir^n.

$$\frac{\mu_l}{l} = \frac{e}{2m_e}$$

This ratio $\frac{\mu_l}{l}$ is called gyromagnetic ratio.

It is constant and $\frac{\mu_l}{l} = 8.8 \times 10^{10}\ C/kg$ for an electron.

According to Bohr model

$$l = \frac{nh}{2\pi} \qquad n = 1, 2, 3 - - - .$$

$h \rightarrow$ Planck's constant

This discreateness is called - Bohr quantisation condition

Now $$\mu_l = \frac{e}{2m_e} \cdot l$$

$$= \frac{e}{2m_e} \cdot \frac{nh}{2\pi}$$

Put $n = 1$, $h = 6.626 \times 10^{-34}\ JS^{-1}$
$m_e = 9.11 \times 10^{-31}\ kg$

we get

$$(\mu_l)_{min} = 9.27 \times 10^{-24}\ Am^2$$

This value is called the Bohr magneton.

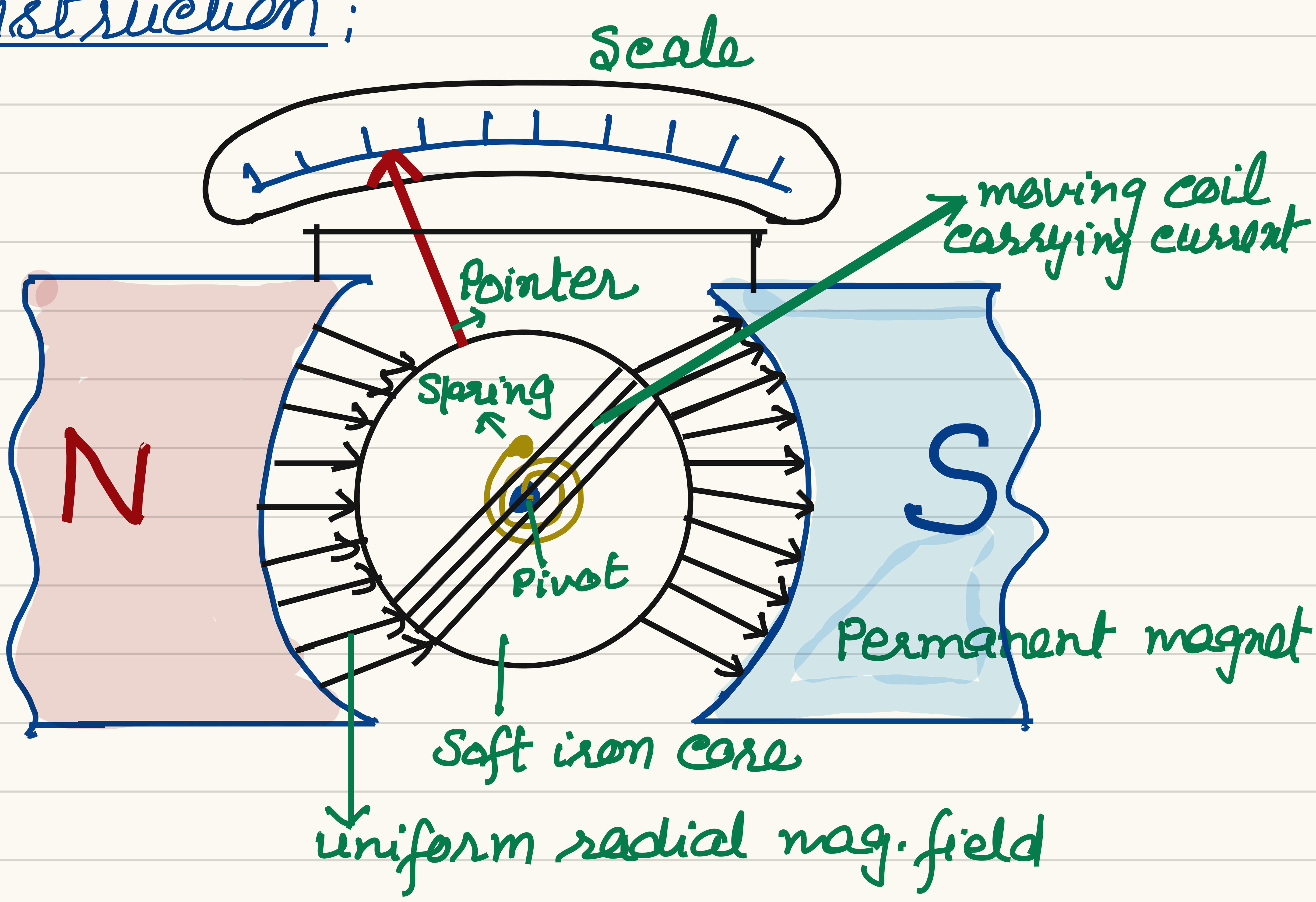
* Besides the orbital moment, the electron has an intrinsic magnetic moment which is called spin magnetic moment and has the same numerical value.

Moving coil Galvanometer (MCG)

This is an instrument used for detection and measurement of small electric current.

<u>Principle</u>: When a current carrying coil is placed in magnetic field it experience a torque.

<u>Construction</u>:



A Weston galvanometer consist of a rectangular coil of fine insulated copper wire wound on a light non magnetic (Al) frame. The two ends of circle of this frame are pivoted b/w two jewelled bearings. The motion is contralled by a pair of hair springs of phosphor bronze. The spring provided restoring torque. A pointer is attached to the coil to measure its deflection on a suitable scale.

A soft iron core is mounted b/w magnet. This makes the mag. field radial. Soft iron core increases the strength of magnetic field.

<u>Working:</u> When current flows in the coil a torque is set up. In equilibrium –

Deflecting torque τ_m = Restoring torque τ_R

$$NIABSin\theta = K\phi$$

here

$K \rightarrow$ Torsional constant

$\phi \rightarrow$ twist produced

for $\theta = 90°$

$$NIAB = K\phi$$

$$\phi = \left(\frac{NAB}{K}\right) I$$

or $\phi \propto I$

and $I = \left(\frac{K}{NAB}\right)\phi = G\phi$

where

$G = \frac{K}{NBA}$ = Galvanometer constant (current reduction factor)

<u>Figure of Marit:</u>

The ratio of current and the angle of deflection is called figure of marit.

$$G = \frac{I}{\alpha} = \frac{K}{NBA}$$

<u>Sensitivity of Galvanometer</u>:

A galvanometer is said to be sensitive if it shows large scale deflection even for small current.

<u>Current Sensitivity</u>: It is defined as the deflection per unit current.

$$I_S = \frac{\alpha}{I} = \frac{NAB}{K}$$

<u>Voltage Sensitivity</u>: It is defined as the deflection per unit voltage.

$$V_S = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NAB}{KR}$$

Clearly $V_S = \frac{I_S}{R}$

* current sensitivity can be increased by -
  - increasing N, A and B
  - decreasing K

* Increasing I_S may not necessarily increase the voltage sensitivity. because

if $N \rightarrow 2N$

$I_S \rightarrow \frac{\phi}{I} \rightarrow 2\frac{\phi}{I}$ [If no. turns doubled, I_S also gets doubled]

but if $N \rightarrow 2N$
$R \rightarrow 2R$

thus $V_S \rightarrow \frac{\phi}{V} \rightarrow \frac{\phi}{V}$ $\left\{ V_S = \frac{\cancel{2}NAB}{K(\cancel{2}R)} = \frac{NAB}{R} \right.$

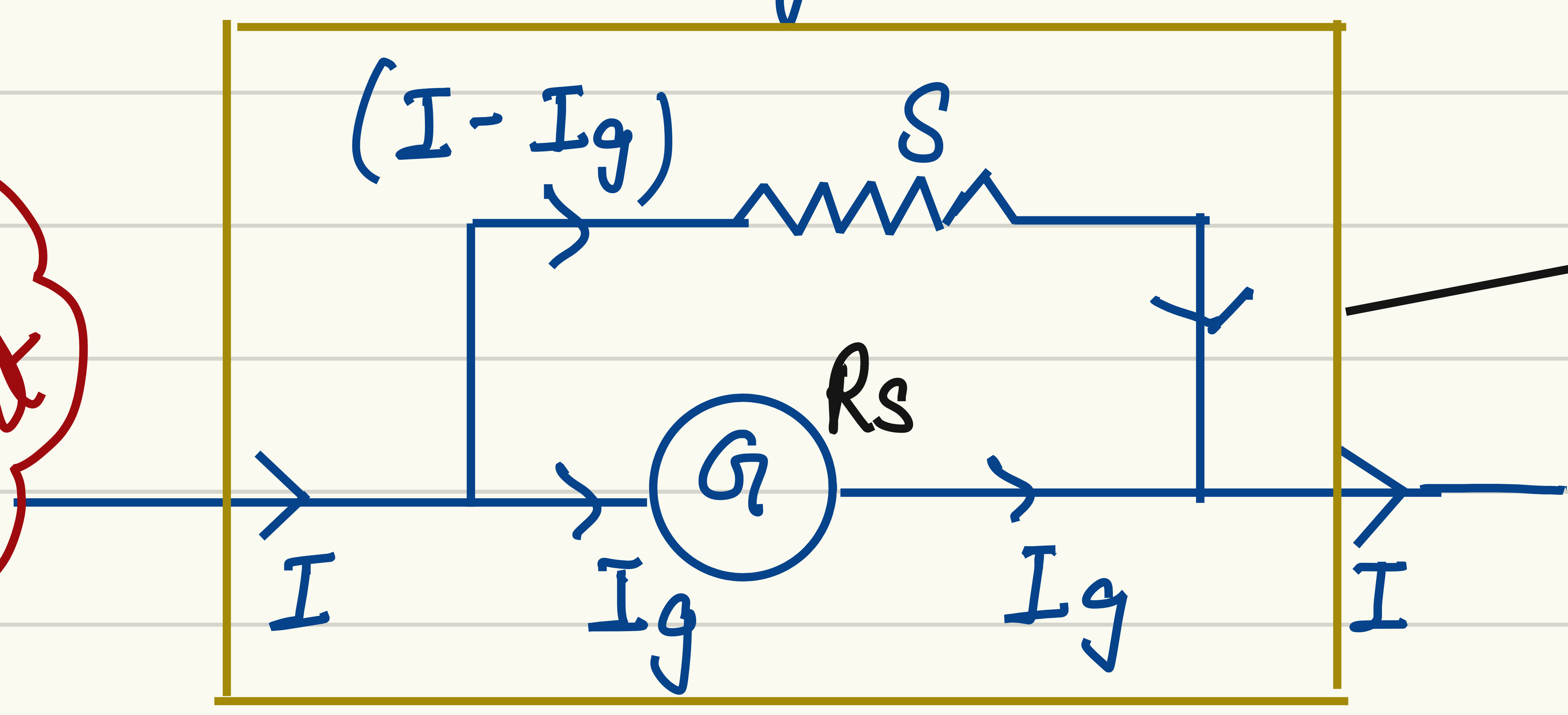
V_S remain unchanged on incrasing N. but can be incrase by incrasing A & B and by decrasing K.

Conversion of Galvanometer Into Ammeter and Voltmeter

(1) **Galvanometer as an Ammeter:**

A galvanometer is converted into an ammeter by connecting a low resistance (shunt S) in parallel with galvanometer.

Ammeter is used to measure current so it has low resistance



→ Ammeter

$R_s \rightarrow$ Resistance of G

$$V_{galvanometer} = V_{shunt}$$

$I_g \rightarrow$ Current in galv.

$G \rightarrow$ Resistance of galvanometer

$$I_g R_g = (I - I_g) S$$

or
$$\boxed{S = \frac{I_g R_g}{(I - I_g)}}$$

or
$$(I - I_g) = I_g \frac{R_g}{S}$$

or
$$I = I_g \frac{R_g}{S} + I_g$$

or
$$I = I_g \left(\frac{R_g + S}{S} \right)$$

or
$$I_g = \left(\frac{S}{R_g + S} \right) I$$

An ammeter has low resistance and it is always connected in series to the circuit

i.e.
$$\boxed{I_g \propto I}$$

* To increase the range of ammeter n times shunt S to be connected as $S = G/(n-1)$

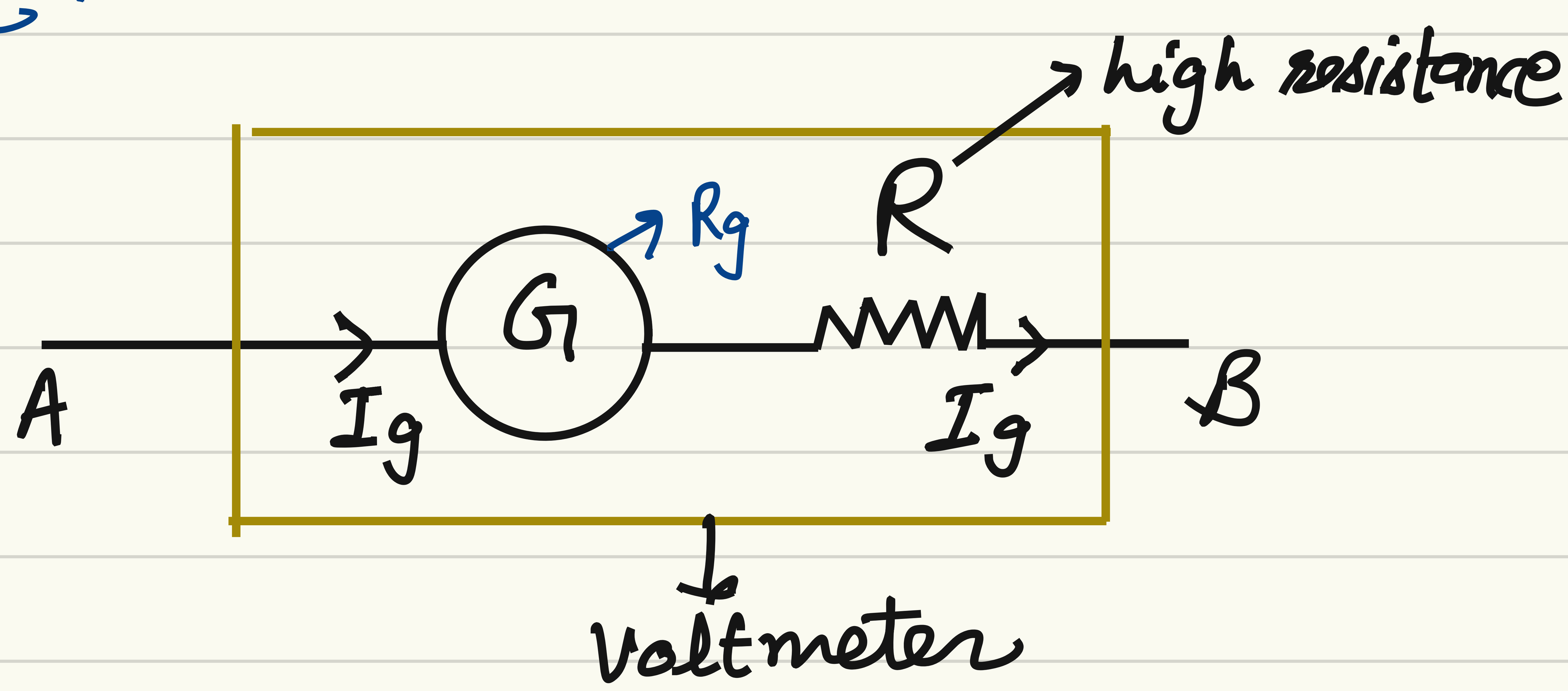
* Shunt S is connected in parallel to galvanometer. Therefore

$$\frac{1}{R_{eff}} = \frac{1}{R_g} + \frac{1}{S} \Rightarrow R_{eff} = \frac{R_g S}{R_g + S}$$

* An ideal ammeter has zero resistance. Therefore to convert the galvanometer into ammeter, it is connected in parallel to reduce the R_{eff}.

Galvanometer as a Voltmeter:

A galvanometer is converted into voltmeter by connecting high resistance R is series with galvanometer.



In series,

$$I = I_g$$

$$= \frac{V}{R_{eff}}$$

$V \rightarrow$ Potential difference

or $$I = \frac{V}{R + R_g}$$

$[R_{eff} = R + R_g]$

or $$R + R_g = \frac{V}{I}$$

or $$\boxed{R = \frac{V}{I} - R_g}$$

here $I_g \propto V$

* Since the galvanometer and high resistance R are connected in series

$$R_{eff} = R + R_g$$

* An ideal voltmeter has infinite resistance.
* Resistance of voltmeter is very high, so it draw least current.
* Voltmeter is always connected in parallel with circuit element through the potential is to be calculated.
* In order to increase the range of voltmeter n times, the resistance to be connected in series with G is

$$R = (n-1)\,G$$