

Alternating Current

1. Alternating Emf

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t \quad \text{or} \quad \mathcal{E} = \mathcal{E}_0 \cos \omega t$$

$\mathcal{E}_0 \rightarrow$ maximum value of emf

$\omega \rightarrow$ Angular frequency

$\omega = 2\pi f$, where f is frequency

2. Average value of AC current and AC voltage for one complete cycle

$$(I_{av})_{1 \text{ cycle}} = 0$$

and $(\mathcal{E}_{av})_{1 \text{ cycle}} = 0$

3. Average value of AC current and voltage for half cycle

$$(I_{av})_{\text{half}} = \frac{2}{\pi} I_0 = 0.637 I_0$$

and $(\mathcal{E}_{av})_{\text{half}} = \frac{2}{\pi} \mathcal{E}_0 = 0.637 \mathcal{E}_0$

4. Root mean square current (I_{rms} or I_{eff} .)

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

also,

$$I_{rms} = \sqrt{\frac{I_1^2 + I_2^2 + \dots + I_n^2}{n}}$$

$$= \sqrt{\frac{1}{T} \int_0^T I^2 dt}$$

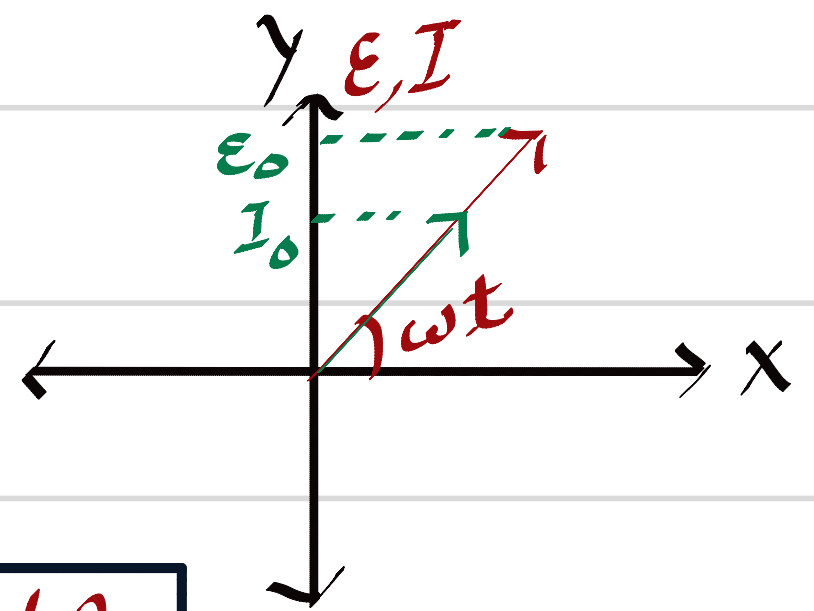
5. AC circuit containing Resistor Only

$$\text{Emf } \mathcal{E} = \mathcal{E}_0 \sin \omega t$$

$$\text{current } I = I_0 \sin \omega t$$

$$\text{Resistance } R = \frac{\mathcal{E}}{I} = \frac{\mathcal{E}_0}{I_0}$$

* $\mathcal{E}$ and I are in same phase

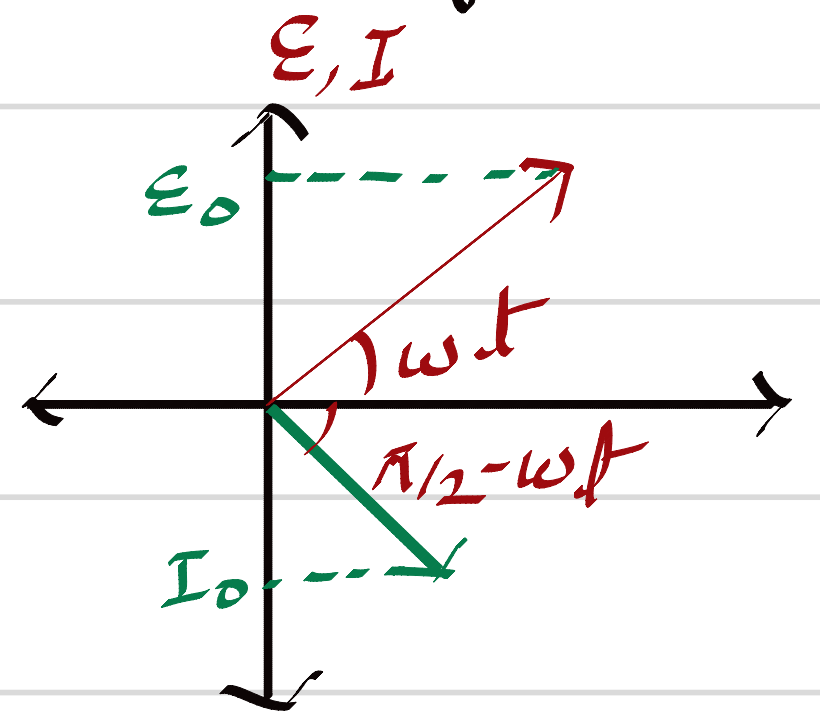


6. AC circuit containing Inductor Only

$$\text{Emf } \mathcal{E} = \mathcal{E}_0 \sin \omega t$$

$$\text{current } I = I_0 \sin \left(\omega t - \frac{\pi}{2} \right)$$

* I lags behind $\mathcal{E}$ by $\frac{\pi}{2}$



Inductive Reactance (X_L)

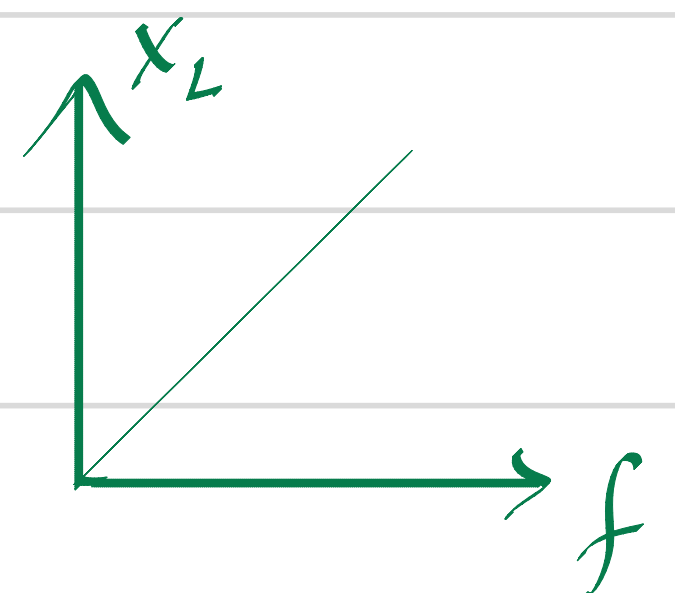
$$X_L = \omega L = 2\pi f L$$

SI unit of X_L is Ohm

here $\omega \rightarrow$ angular frequency in rad s^{-1}

$f \rightarrow$ frequency in rev. s^{-1} or Hz

$L \rightarrow$ Inductance in henry

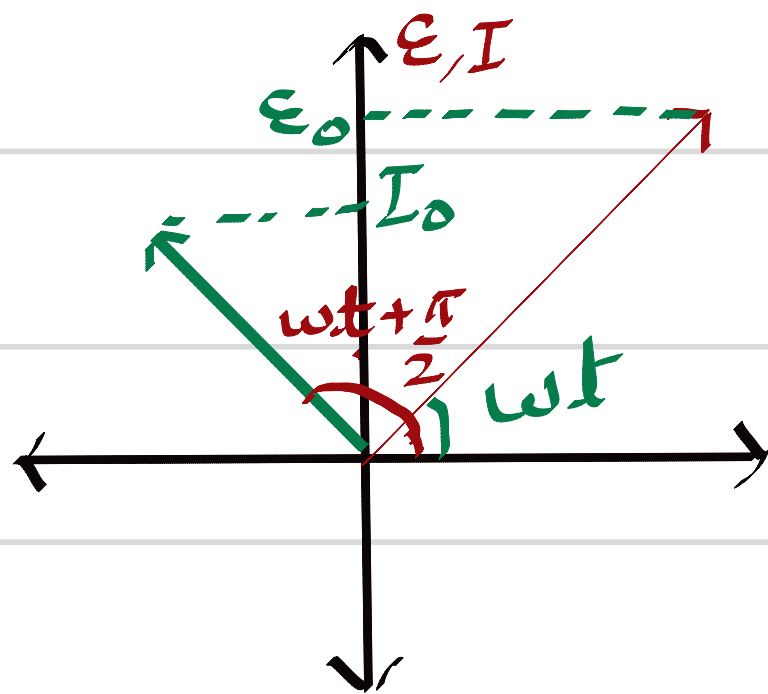


7. AC circuit containing capacitor only

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

$$I = I_0 \sin(\omega t + \frac{\pi}{2})$$

* Current I is leading emf $\mathcal{E}$ by $\frac{\pi}{2}$



Capacitive Reactance (X_C)

$$X_C = \frac{1}{\omega C}$$

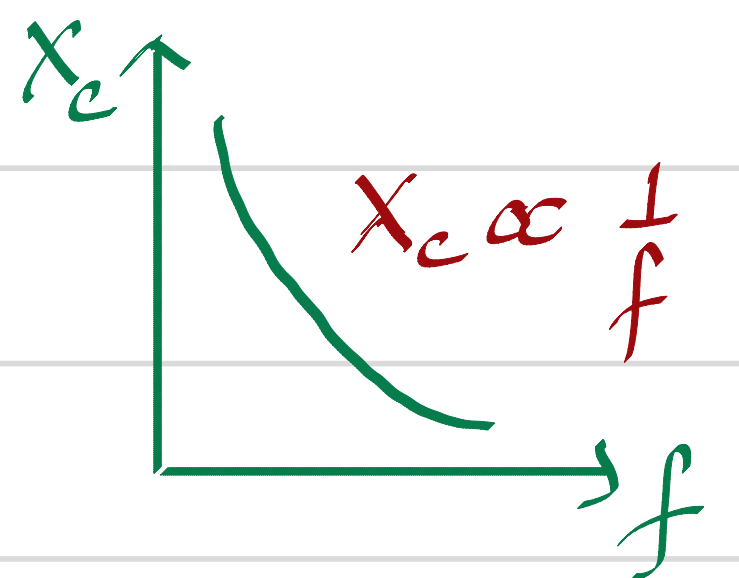
$C \rightarrow$ capacitance in Farad

$$= \frac{1}{2\pi f C}$$

$$[\omega = 2\pi f]$$

* For AC $X_C \propto \frac{1}{f}$

* For DC $X_C = \infty$ as $f = 0$



SI unit of X_C is ohm

8. AC circuit containing Resistor and Inductor in series (Series RL circuit)

$$\text{Emf } \mathcal{E} = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I \sqrt{R^2 + X_L^2}$$

$$\mathcal{E} = I \sqrt{R^2 + (\omega L)^2}$$

$$[V_R = IR, V_L = IX_L]$$

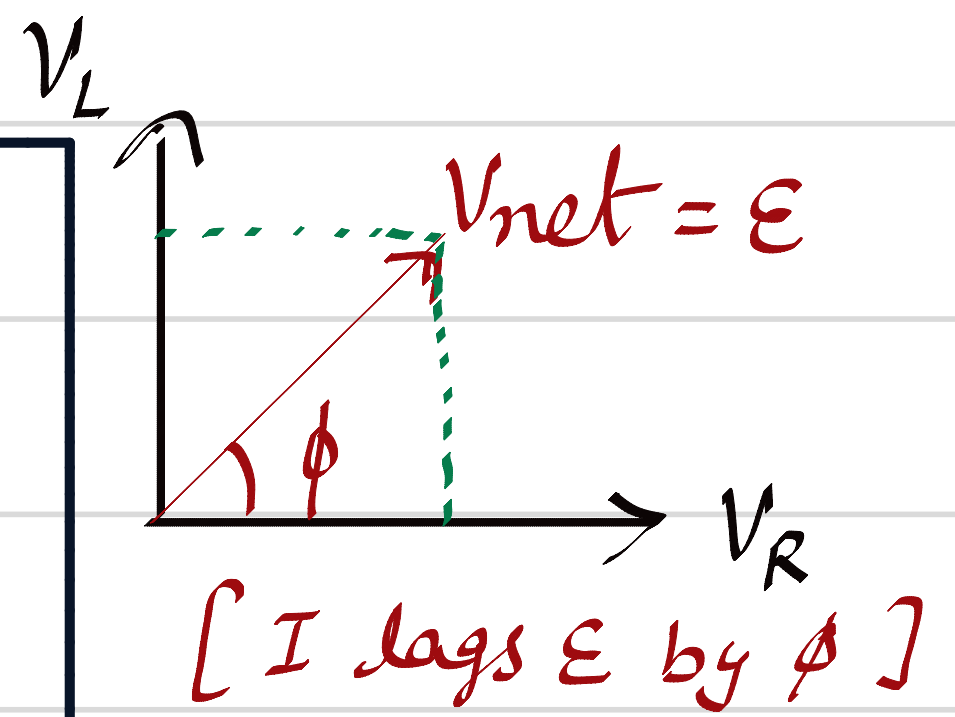
$$[\omega = 2\pi f]$$

→ Impedance

$$\frac{\mathcal{E}}{I} = Z = \sqrt{R^2 + (\omega L)^2}$$

$$\text{or } I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + X_L^2}}$$

$$\text{and } \tan \phi = \frac{V_L}{V_R} = \frac{X_L}{R}$$



9. Series RC circuit

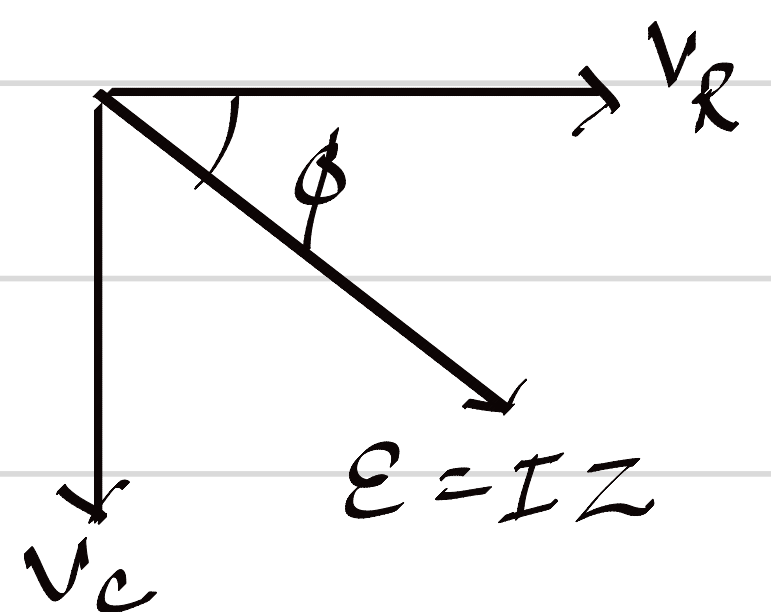
$$\begin{aligned} \mathcal{E} &= \sqrt{V_R^2 + V_C^2} \\ &= \sqrt{(IR)^2 + (IX_C)^2} \quad [V_R = IR, V_C = IX_C] \\ &= I \sqrt{R^2 + X_C^2} \\ &= I \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \quad [X_C = \frac{1}{\omega C}, \omega = 2\pi f] \end{aligned}$$

$$\text{or } \frac{\mathcal{E}}{I} = Z = \sqrt{R^2 + X_C^2}$$

$$I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + X_C^2}}$$

SI unit of
Z is ohm

$$\tan \phi = \frac{V_C}{V_R} = \frac{X_C}{R}$$



[I leads $\mathcal{E}$ by ϕ]

10. Series LC R circuit

(i) When $V_L > V_C$

$$\begin{aligned} \mathcal{E} &= \sqrt{V_R^2 + (V_L - V_C)^2} \\ \mathcal{E} &= I \sqrt{R^2 + (X_L - X_C)^2} \end{aligned}$$

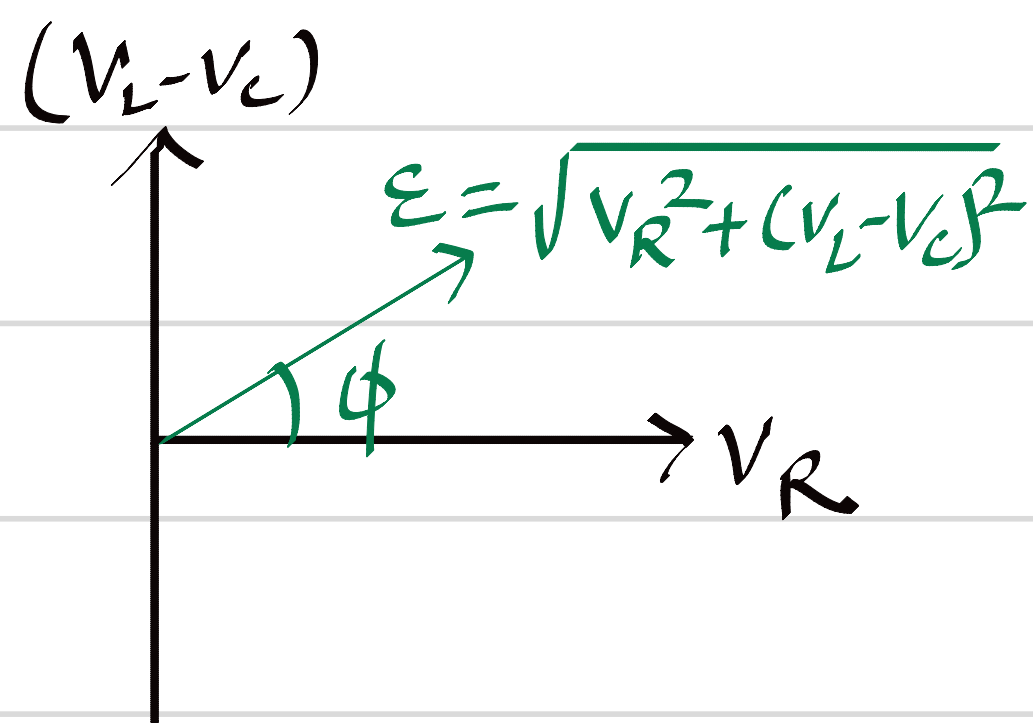
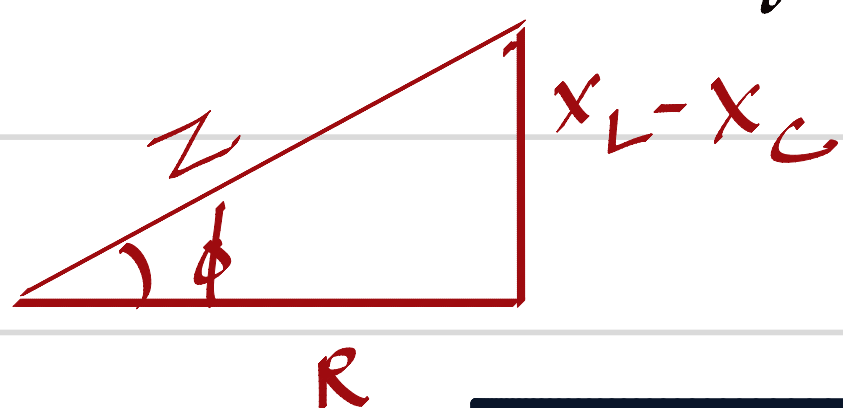
$$\frac{\mathcal{E}}{I} = Z = \sqrt{R^2 + (X_L - X_C)^2}$$

or $I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

Z is impedance

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Impedance Triangle



$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

(i) When $V_C > V_L$

$$I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (X_C - X_L)^2}} = \frac{\mathcal{E}}{\sqrt{R^2 + (\frac{1}{\omega C} - \omega L)^2}}$$

$$\tan \phi = \frac{X_C - X_L}{R}$$

(ii) Resonance condition ($X_L = X_C$)

When $X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$

$$\omega^2 = \frac{1}{LC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Resonance frequency

$$[\omega = 2\pi f]$$

11.

Power factor ($\cos \phi$)

$$\cos \phi = \frac{P}{E_{rms} I_{rms}}$$

 $P \rightarrow$ True Power

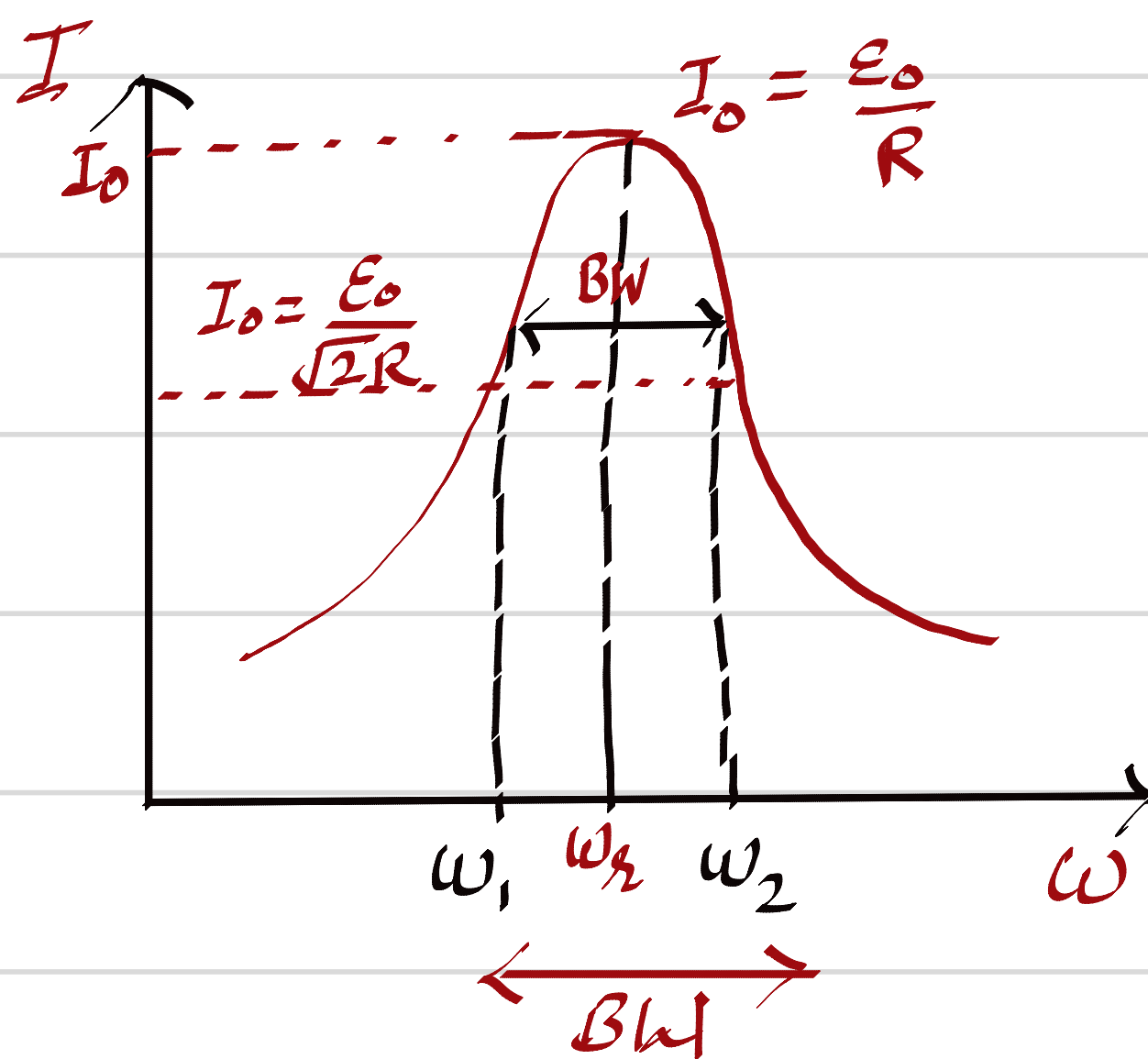
* Its value is b/w 0 to 1

$$\cos \phi = \frac{R}{Z}$$

12.

Quality factor (Q-factor)

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{2\Delta\omega} = \frac{\text{Resonance frequency}}{\text{Bandwidth}}$$

(more I , less R)

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13.

Power in AC circuit

$$P = \frac{1}{2} \epsilon_0 I_0 \cos \phi$$

$$P = E_{rms} I_{rms} \cos \phi$$

Cases -

(i) For Pure Resistive circuit

$$P = VI = \frac{V^2}{R} = I^2 R \quad (\text{In watts})$$

(ii) For Pure capacitive or Pure inductive circuit

$$P = 0 \quad [\text{Wattless current}]$$

(iii) For LCR circuit

$$P = \frac{E_0 I_0 \cos \phi}{2} = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi$$

$$P = E_{rms} I_{rms} \cos \phi \quad \left[\cos \phi = \frac{R}{Z} \right]$$

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Transformer

$$E_1 = -N_1 \frac{d\phi}{dt}, \quad E_2 = -N_2 \frac{d\phi}{dt}$$

N_1 and N_2 are no. of turns in primary and the secondary coil

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

(For ideal transformer $P = E_1 I_1 = E_2 I_2$)

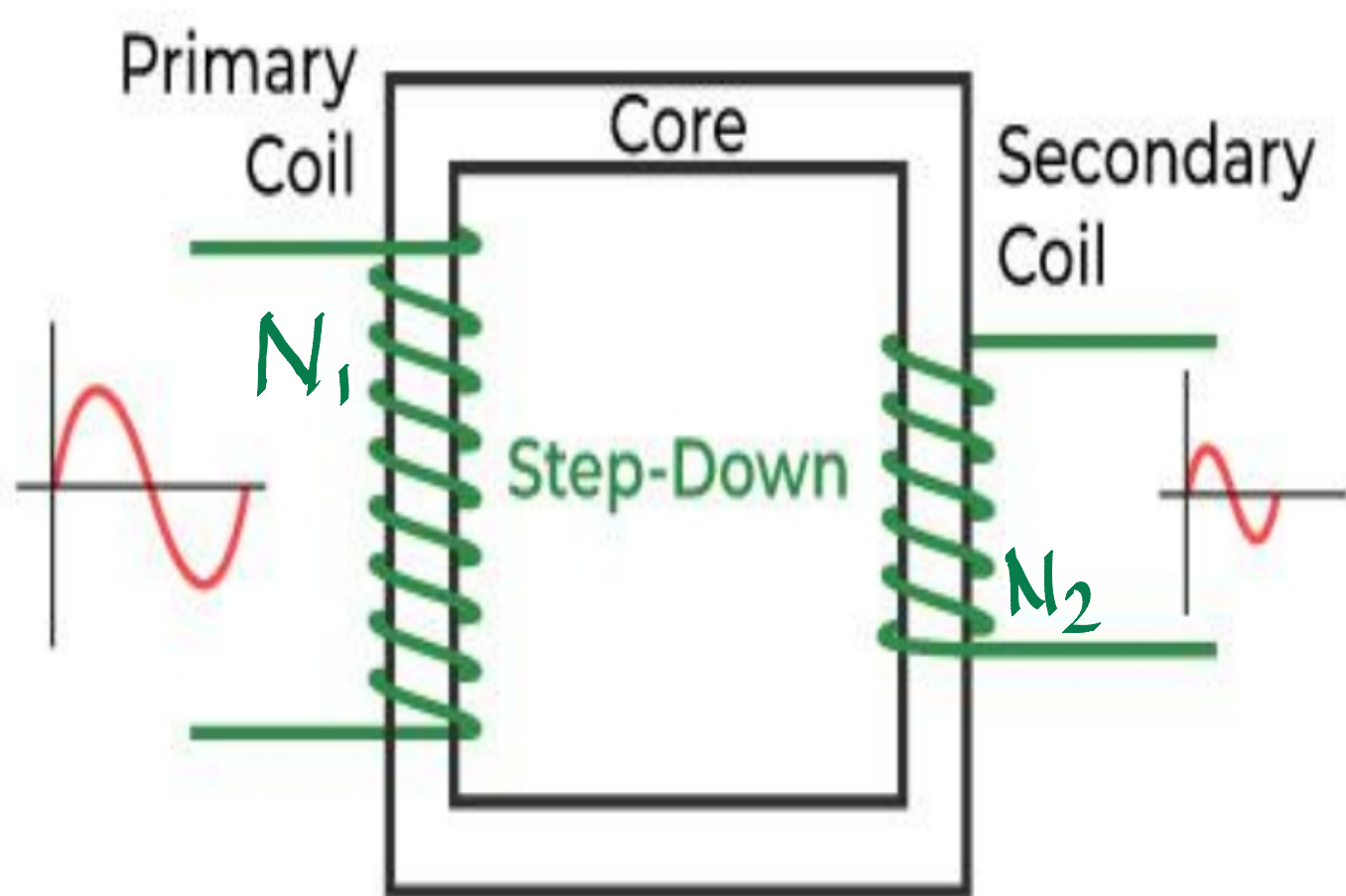
Efficiency of transformer

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100$$

Flux linked with the coil

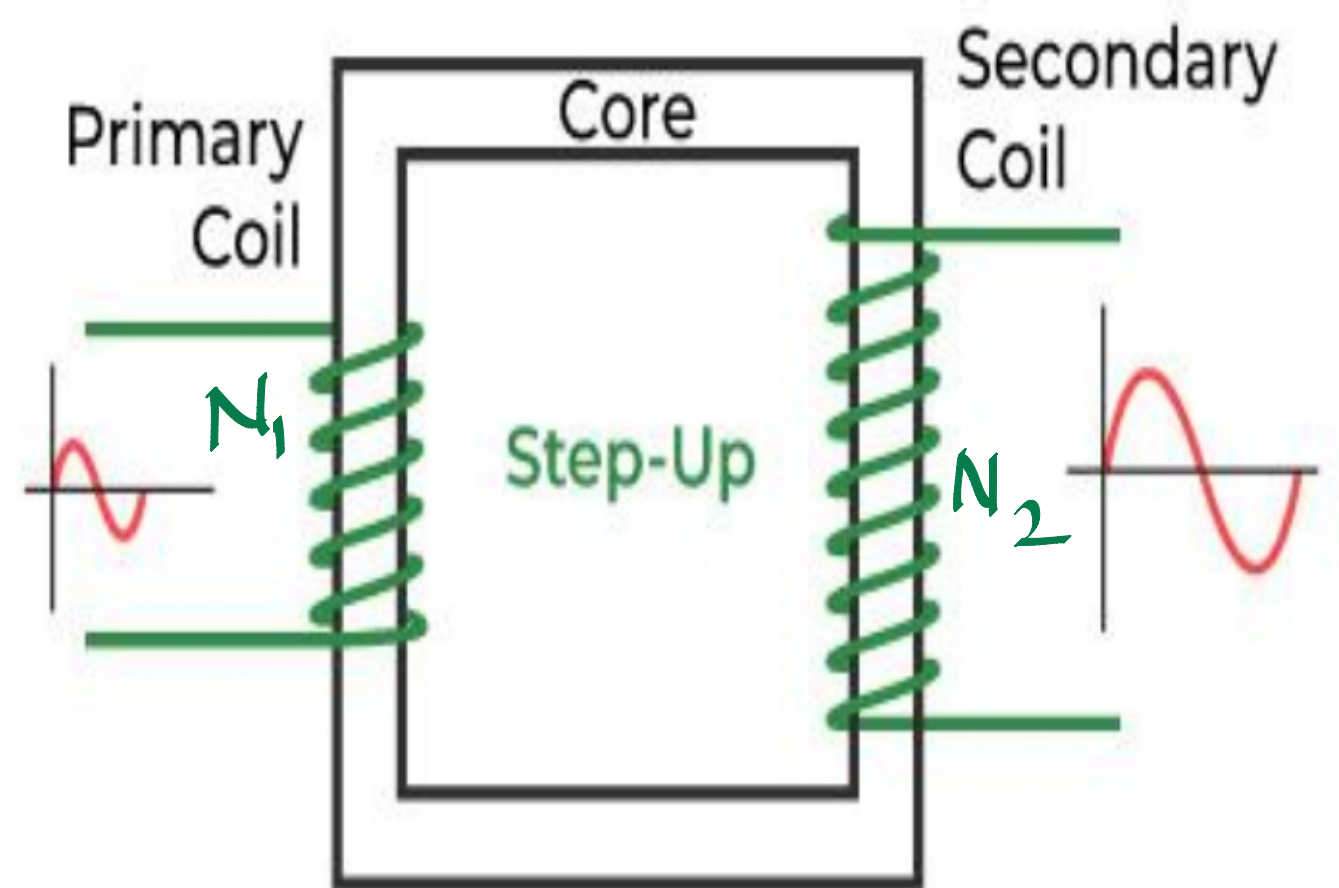
$$\phi = NBA \cos \omega t$$

Types of Transformer



Step-Down Transformer

$$N_2 < N_1$$



Step-Up Transformer

$$N_2 > N_1$$